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The Competition Between ISPs in Presence of the Net Neutrality

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ABSTRACT

In this work, we present an economic model of computer networks that describes the in-teraction between Internet Service Providers (ISP), customers and content provider. The competition between ISP s may be translated by the prices they require and the qualities of service (QoS) they offer. The customer demand for service from an ISP does not only de-pend on the price and quality of service (QoS) of the ISP , but it is influenced by all those offered by its competitors. This behavior has been extensively analyzed using game the-ory as a decision support tool. We interpret a non-neutral network when a content provider privileges ISP s by offering them more bandwidth to ensure proper QoS to support ap-plications that require more data transport capacity (voice over internet protocol (V OIP) the live video streaming, online gaming). In addition, our work focuses on the price game analysis and QoS between ISP s in two cases: neutral network and non-neutral network. After showing the existence and uniqueness of equilibrium in terms of quality of service, we analyzed the impact of net neutrality on competition between ISP s. We also validated our theoretical study with numerical results, which show that the game has an equilibrium point which depends on all the parameters of the system.

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1. INTRODUCTION

Currently Net neutrality is a major issue. This principle is one of the founders of the Internet, enabling excludes any discrimination of data transmitted over the network. This concept is not clearly defined, has aroused great debate in many different places such as the economy, academia, law, Internet and congress industry. Consequently, net neutrality has become an attractive cross-cutting issue requiring the aggregation of the effort of a huge scientific community from different disciplines. The lack of rigorous but simple models, and as complete as possible in the related documentation were the main motivations for this work. The neutrality of network in the long term was introduced as a result of the concept of common support. The two most commonly potential behaviors cited are the ability of network providers to control access and pricing of broadband facilities and incentives to promote the network of owned content, placing unaffiliated content providers at a competitive disadvantage [1].

In this paper, we focus on the impact of the second conduct on competition between internet service providers. Several works addressed the issue of network neutrality from different aspects [2] [3], [4], [5], [6]. [1] provides a new and comprehensive insight into the context of the debate on net neutrality. The contributions that are most related to ours are [2], [5], [7], [8] and [9]. In this paper, we consider several providers share a market and competition among providers occurs in prices and in terms of the quality of service they offer. We assume that the demand for the service of a particular internet service provider is a function of the vector of price and quality of service of all ISP s. We establish the existence and uniqueness of Nash equilibrium for both plans of neutrality or non-neutrality. We analyze the effects of net neutrality on the invoice price and QoS offered as the behavior of internet service providers.

The rest of the paper is organized as follows. In Section 2., we describe the model of discord between internet service providers and their subscribers, also, we present definition of Nash equilibrium. In Sections 3. and 4. we provide theorems for existence and uniqueness of equilibria respectively in case of net neutrality and non-neutrality. Section 5. presents numerical study to validate our claims, Section 6. presents a study of the impact of competition and Section 7. concludes the paper.

2. PROPLEM MODELING

Our economic model is composed of a content provider *(CP)* and *N* Internet Service Providers *(ISPs)* in competition on a set of customers. The figure 1 illustrates a typical example of systems with a single content providern*(CP)* and several Internet Service providers which each serves a set of end users *(EU)*. Here, the *CP* provide multimedia contents (eg, music streaming, *V OIP*, ...). This multimedia content is transported and placed at the disposal of the *EUs* on the physical infrastructure of ISPi. Under the neutral network configuration, *EUs* and the content provider pay only for their direct access. On one hand, and according to the rate of the own demand for *ISPi*, *CP* decided to invest and allocate bandwidth _ic to it. This can be considered as a preferred contract is between *ISPi* and *CP*. Yet, as the demand of *ISPi* increases as it becomes more beneficial to *CP* to invest more and more. On the other, each *ISPi* charges to its customers an amount pi per unit of traffic. Moreover, each *ISPi* allocates a bandwidth _i to its customers to guarantee their promised *QoS* *~qi*. In the rest, we consider the following notations: $p = (P_1, ..., P_j, ..)$ and $\tilde{q} = (\tilde{Q}_1, ..., \tilde{Q}_j, ...)$ for the charged price vector and the promised *QoS* respectively.

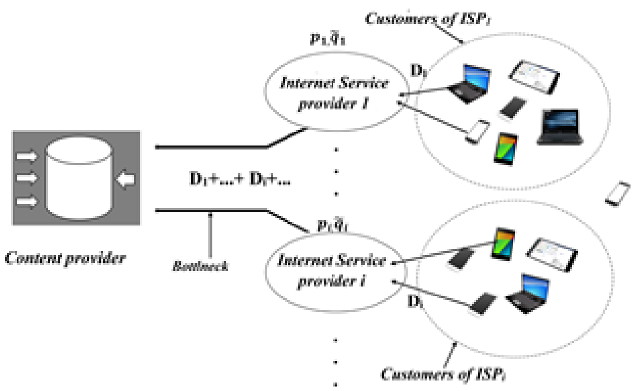

Figure 1. Single content provider and several internet service providers

2.1. End-to-end Quality of Service (QoS)

The quality of service (QoS) is the ability to transmit in good conditions a given type of traffic, in terms of availability, throughput, transmission delay, jitter, packet loss rate ... The guarantees of the quality of service are important if the network capacity is very limited, especially for real-time streaming of multimedia applications such as Voice over IP (V OIP), online gaming and IP -T V . This last reason encourages users to subscribe to an ISP promising a good quality of service. Obviously, the end-to-end (e2e) QoS depends on investments of ISP s and CP . We claim that e2e QoS implicitly depends on the demand Di(:) for ISPi services, the amount of bandwidth reserved i by ISPi for EU s and the amount of bandwidth reserved ic by the content provider to this ISP . In other words, the e2e QoS depends on the perceived quality on both links EU-ISP and ISP -CP . Certainly, the end-to-end quality of service can express the transmission end-to-end delay, the end-to-end throughput, the end-to-end loss probability or any combination of many quality of service indicators.

Later, we will be limited to certain special cases where the end-to-end quality of service is simple enough (eg, the end-to-end delay is the sum of delays experienced on both links EU-ISP and ISP -CP).

2.2. Demand Model

We consider that the demand function Di(:) for the ISP services is linear with respect to price (pi) fixed by the ISPi and promised e2e QoS, see [10]. This demand also depends on price p i and e2e QoS of competitors, namely, the demand function depends on p and q~. Naturally, the demand function Di is decreasing w.r.t pi and increased w.r.t p i, while it is increasing w.r.t q~i and it is decreasing, w.r.t q~ i. We consider that the demand functions w.r.t the ISPi service can be written as follows:

$$D_i(p,\tilde{q}) = a_i - \alpha_i^i p_i + \beta_i^i \tilde{q}_i + \sum_{j,j\neq i} \left[\alpha_i^j p_j - \beta_i^j \tilde{q}_j\right], \tag{1}$$

where α_i is a positive constant used to ensure the non-negativity of the demands on the feasible region.

Considering that α_i^j and β_i^j are positive constants and standardized, which represent respectively the the ISP_i sensitivity to the price and the QoS of the ISP_j, as follows:

$$\sum_{j=1}^{N} \alpha_i^j = 1 \;\; and \;\; \sum_{j=1}^{N} \beta_i^j = 1 \;\; , \qquad i = 1, ..., N.$$

Assumption 1 *For any e2e QoS profile, the e2e QoS mutual sensitivities satisfy:*

$$\beta_i^i \geqslant \sum_{j,j\neq i} \beta_i^j \;\; , \;\; \forall\, i,j = 1,...,N. \tag{2}$$

The assumption 1 is realistic and has no impact on the field of applicability of this work. Indeed, considering that the influence of *e2e QoS* promised by the *ISPi* on its gains more weight relative to the sum of the influences of promised *e2e QoS* by ISPs competitors on *ISPi* gains. This condition could then take into account the presence of loyalties of customers and/or partial knowledge of *e2e QoS* of competitors. Moreover, this assumption is a reasonable condition which guarantees supermodularity of the game and So the uniqueness of Nash equilibrium.

Assumption 2 *For any vector quality of service of competitors, the total demand* $D(p,\tilde{q}) = \sum_{i=1}^{N} D_i(p,\tilde{q})$ *is an in-creasing function w.r.t the individual e2e QoS* $\tilde{q}_i$ *of the* ISP_i.

Considering the desire of paying customers, it becomes plausible to consider that the total demand is in-creasing depending on individual *e2e QoS* fixed by *ISPi*. When the *ISPi* decides to decrease its quality of service, attached customers would be migrated and subscribe with its competitors or decide to unsubscribe. In other words, this assumption says that the effect of the *e2e QoS* individual is still higher than the overall perceived influence from competitors. An important characteristic is that the variation of the total demand, compared to individual *e2e QoS* is very low, because few customers decide to unsubscribe completely.

Remark 1 *From the assumption 2, we deduce that the first derivative of total demand w.r.t* $\tilde{q}_i$ *is positive, then, the e2e QoS mutual sensitivities satisfy:*

$$\beta_i^i - \sum_{j,j\neq i} \beta_i^j \geqslant 0 \;\; , \;\; \forall i,j = 1,...,N. \tag{3}$$

Assumption 3 *We assume that the influence of QoS e2e for an* ISP_i *on its demand is inversely proportional to the number of its competitors (N).*

$$\beta_i^i - \sum_{j,j\neq i} \beta_i^j \leqslant \frac{1}{N} \;\; , \;\; \forall i,j = 1,...,N. \tag{4}$$

The number of competitors can influence the demand for a particular *ISP*$_i$ in a remarkable manner when the number becomes bigger and vice versa, i.e. of the sum of the mutual sensitivities to its competitors quality of service become very close to that of the *ISP*$_i$, so that deference sensitivities can be bounded by the inverse of the total number of these competitors.

Remark 2 *From the assumption 3 and remark 1, we can say:*

$$0 \leqslant \beta_i^i - \sum_{j,j\neq i} \beta_i^j \leqslant \frac{1}{N} \;\; , \;\; \forall i,j = 1,...,N. \tag{5}$$

2.3. Utility Model

We turn now to take the utility function of each *ISP s* and the single *CP* . Let qi (end-to-end quality of service) the real quality perceived by the *EUs* of *ISPi*, (In this case q~i is the *QoS* promised / advertised by the *ISPi* for end users). The net revenue of *ISPi* is exactly the difference between its total revenue and its expenses. These latter correspond to the sum of the costs to cover the costs of bandwidth i and a certain penalty if it does not meet promised quality *qi*. Thus, net revenue is given by as following:

$$U_i^{ISP}(p,\tilde{q}) = p_i D_i(p,\tilde{q}) - \vartheta_i \phi_i + \theta_i(q_i - \tilde{q}_i), \tag{6}$$

where ϑ_i and θ_i are positive constants.

2.4. Game Analysis

In the rest of this work, we will analyze the game that arises in both the establishment of neutrality and non-neutrality. The players are the *ISP s* that must to define their strategies for the price *(pi)* and the promised quality of service *(qi)*. The concept of neutrality is considered assuming that the *CP* distributes its bandwidth as the pro-cessor sharing principle. In another hand, the non-neutrality scenario is taken into account by assuming that there is one (more) specific *ISP (s)* (namely *ISP1*) who signed a contract with the *CP* , in order to reserve him a amount of bandwidth to ensure the promised quality of service *(q~1)*. In this context, the competitors of *ISP1* equally share the remaining bandwidth as if we have neutrality case by taking into account the topology when we do not have the *ISP1*.

Definition of Nash equilibrium of the QoS game: We consider a game of strategic form of N-players

$$\Gamma = \{\mathcal{N}, \tilde{Q}_1, ..., \tilde{Q}_N, U_1^{ISP}, ..., U_N^{ISP}\}, \tag{7}$$

where $\tilde{Q}_i$ is the set of QoS strategies of player i and U_i^{ISP} its utility function.

Definition 1 *Nash equilibrium specifies a strategy $\tilde{q}_i{}^* \in \tilde{Q}_i$ for each player i (with $i = 1, ..., N$) in such a way that:*

$$U_i^{ISP}(p,\tilde{q}^*) = \max_{\tilde{q}_i \in \tilde{Q}_i} U(p, \tilde{q}_1{}^*, ..., \tilde{q}^*_{i-1}, \tilde{q}_i, \tilde{q}^*_{i+1}, ..., \tilde{q}^*_N), \tag{8}$$

when the vector of price parameters, p, of all providers is fixed to a certain predetermined point.

Below, we analyze the competitive qualities of service for *N ISPs* that maximize their utilities. To do so, we demonstrate the existence and uniqueness of the game equilibrium between *N ISPs*, after we calculate the equilibrium point. To analyze equilibrium of the game, we need to find properties on the utility function which require that we describe the expression in both cases.

2.5. Learning Nash equilibrium

In [8, 3, 7, 10], the main concern was the search for the equilibrium situations (namely Nash). Lately researchers are questioning the need for convergence of a learning algorithm to a Nash equilibrium, there are several reasons for this. First, there may the multiple equilibria in a game, and it can not any method for coordinating choice of agent.

Learning algorithms resemble the behavior of competitors in many naturally arising games, and thus results, on the convergence or non-convergence properties of such dynamics may inform our understanding of the applicability of Nash equilibria as a plausible solution concept in some settings. In the reality, when every *ISP* tries to maximize its revenue, it is the most natural to accept Nash equilibrium as the optimal solution concept. In Nash equilibrium, each *ISP* 's strategy is the best response to the other *ISP* s's strategies. Thus no *ISP* can gain from unilateral deviation.

Algorithm 1 Best Response algorithm

1: Initialization of QoS vectors;
2: For each ISP_i $i \in \mathcal{N}$ at iteration t:

- $\tilde{q}_i{}^{t+1} = \underset{\tilde{q}_i \in \tilde{Q}_i}{\text{argmax}} \left(U_i(p, \tilde{q}^t)\right)$

3. NEUTRAL NETWORKS

Remember that for the sharing capability of *ISP*s under the Process Sharing (*P S*) principles, ϕ_i is given by combining both the expression of Delay in the links between ISPi and EUs (L. Kleinrock [11])

$$Delay_i^u = \frac{1}{\Phi_i - D_i(p,\tilde{q})},$$

and the expression of the quality of service promised by the ISP_i to end users (EUs)

$$\tilde{q}_i = \frac{1}{Delay_i^u + c_i},$$

Thus:

$$\Phi_i = D_i(p,\tilde{q}) + \frac{\tilde{q}_i}{1-\tilde{q}_i c_i}, \tag{9}$$

where c_i is the expected delay in the link between *ISPi* and *CP* .

The end-to-end delay (denoted $Delay_c^u$) experienced by end users of the *ISPi*, namely, the cumulative delay on both links *EU-ISPi* and *ISPi-CP* , depends on: the total demand for services of, the demand for services transported by ISP_i, $D_i(p,\tilde{q})$, the bandwidth allocated by the CP to ISP_i, Φ_c, and Φ_i the bandwidth allocated by *ISPi* to its customers:

$$Delay_c^u = \frac{1}{\Phi_i - D_i(p,\tilde{q})} + \frac{1}{\Phi_c - D(p,\tilde{q})},$$

The real QoS perceived by end users of the ISP_i is the inverse of the end-to-end delay,

$$q_i = \frac{1}{Delay_c^u} = \frac{\tilde{q}_i(\Phi_c - D(p,\tilde{q}))}{\tilde{q}_i + (1-\tilde{q}_i c_i)(\Phi_c - D(p,\tilde{q}))}, \tag{10}$$

By replacing Φ_i and q_i by their expressions in equation (6), the utility function of ISP_i can be written as:

$$U_i^{ISP}(p,\tilde{q}_i) = (p_i - \vartheta_i)D_i(p,\tilde{q}) - \frac{\vartheta_i \tilde{q}_i}{1-\tilde{q}_i c_i} + \theta_i \tilde{q}_i \left(\frac{\Phi_c - D(p,\tilde{q})}{\tilde{q}_i + (1-\tilde{q}_i c_i)(\Phi_c - D(p,\tilde{q}))} - 1 \right). \tag{11}$$

The question is, under general assumptions, when can we guarantee the existence and uniqueness of the equilibrium due through the *ISP* s? We consider that the quality of service is the only parameter of the game (7) which arose when the price of all *ISP* s is fixed. Thus, under the assumption 1 and according to the remark 1, we have the following general result on uniqueness of quality of service based on Nash equilibrium for all N *ISP* s.

Lemma 1 (Existence) *Considering the game of levels of quality of service which arose when the price vector is fixed for all ISP s, there exists in less one quality of service based Nash equilibrium q~* , of ISP s's game.*

Lemma 2 (Uniqueness) On the assumption 1 and according to the remark 1, the equilibrium point q~* , of the game of ISP s, which arises when the price vector is fixed to all *ISP* s, is unique.

under lemmas (1, 2), we deduce the following theorem:

Theorem 1 (existence and uniqueness) Consider the game of levels of quality of service(7) which arose when the price vector is fixed to all *ISP* s, under the assumptions 1, 3 and according to the remark 2, there is only one level of quality of service based Nash equilibrium q~ of the ISP s's game. (the proof is given by appendices A and B)

4. NON-NEUTRAL NETWORKS

As we mentioned in section 2, in non-neutral networks we assume that there is a specific *ISP* (i.e. *ISP1*) who signed a special contract with *CP* , to reserve for him a quantity of bandwidth to guarantee the promised quality of service *q~1*. Under a non-neutral system, the *ISP1* will guarantee the promised *QoS*, and therefore, the expression of the value of *ISP1* becomes simpler since *q~i = qi i.e.* no penalty does not appear in the function utility of the *ISP1*:

$$U_1^{ISP}(p,\tilde{q}) = (p_1 - \vartheta_1)D_1(p,\tilde{q}) - \frac{\vartheta_1 \tilde{q}_1(\Phi_c - D_1(p,\tilde{q}))}{\Phi_c - D_1(p,\tilde{q}) - \tilde{q}_1}. \quad (12)$$

In this context, the competitors of the ISP1, equally share the remaining bandwidth. In the absence of the ISP1, the network behaves as a neutral network where other ISP s are competing over the common bandwidth. The utility function of other internet service providers (ISP1 competitors) is given by equation (11).

Theorem 2 (existence and uniqueness) Consider the game of quality levels for services (7) that arises without net neutrality. Under the assumption 1 and according to the remark 1, there is one level of Nash equilibrium quality of service therefore the utility function of ISP s satisfies the properties of concavity and uniqueness. (the proof is given by appendix C)

5. NUMERICAL RESULTS

We turn now to discuss how to take advantage of our analytical results. We propose to study numerically the market share game taking account of previous expressions of demand functions and utility of the ISP s. To illustrate, we consider two homogeneous ISP s looking to maximize their respective payoffs. In particular, we discuss the influence of the penalty factor i and the bandwidth of the CP in cases of neutrality and non-neutrality. The figures 2 and 3 represent respectively the curves of the convergence to Nash equilibrium e2e QoS in both neutral and non-neutral network, it is clear that the best response dynamic algorithm 1 convergence to the unique Nash equilibrium e2e QoS, in both cases, we also notice the convergence speed is relatively fast (6 iterations for the neutral case and 5 iterations for the non-neutral case). So this simulation of the algorithm 1 is able to efficiently converge the Nash equilibrium e2e QoS in neutral and non-neutral network.

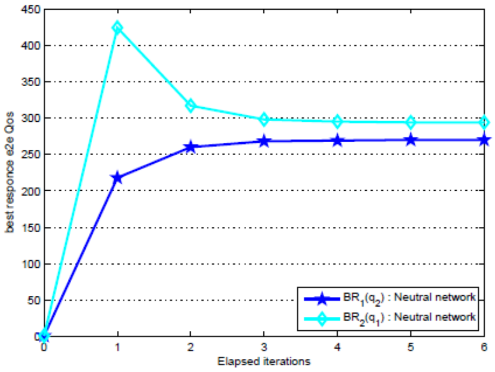

Figure 2. Neutral network: convergence to the Nash equilibrium $e2e\ QoS$

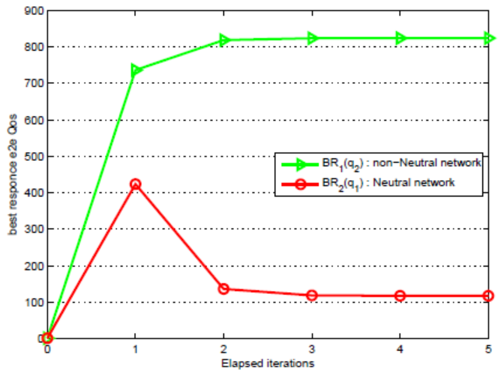

Figure 3. Neutral non-network: convergence to the Nash equilibrium $e2e\ QoS$

5.1. Impact of the QoS penality factor

The figures 4 and 5 represent the variation of the e2e QoS at equilibrium and partitioning of the total demand at equilibrium points w.r.t penalty factor , we note for null values penalty factor, all ISP s, over neutrality or non-neutrality cases, have the same results of e2e QoS (resp. of the demand partitioning). And as we see, the changes in the e2e QoS is similar to those of demand partitioning, which is normal since the demand Di(:) of ISPi is proportional to its e2e QoS (q~i). However the increased penalty factor reduces the e2e QoS (Resp. demand partitioning) which is normal, by that the increase of this factor pushes ISP s in a neutral network to minimize the difference between promised QoS and real QoS, until that ISP s in this regime have for subscribers that these faithful EUs (a=500) for small value of e2e QoS. Thus for non-privileged ISP2, the increasing of this factor has more impact on this ISP compared to ISP s of neutral network, the increase of this factor may even push the EUs of this ISP to migrate to another ISP offering a significant e2e QoS or to unsubscribe completely. These impacts of the increased penalty factor for the non-privileged ISP2 have an inverse impact for the privileged ISP1, which allows it to offer a significant e2e QoS and get a large market share.

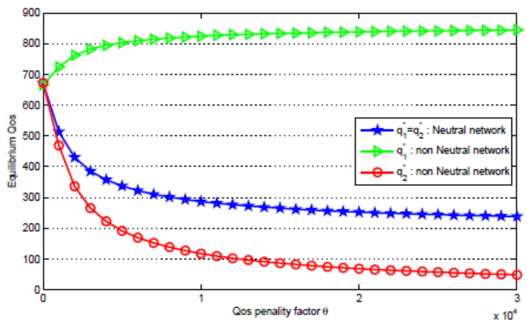

Figure 4. Impact of the penalty factor on equilibrium *QoS*

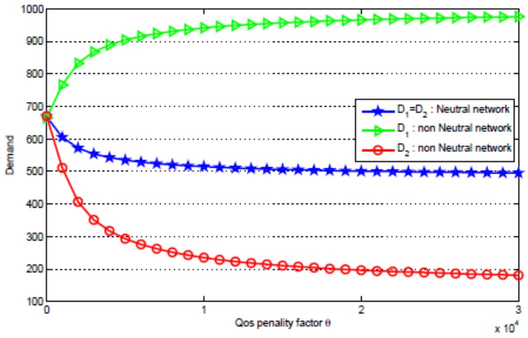

Figure 5. Impact of the penalty factor on Demand partitioning

5.2. Impact of the available bandwidth ϕc

The main text format consist The figure 6 represents the *e2e QoS* variation w.r.t the bandwidth ϕc, we note that for small bandwidth values between (1600 - 3000), *ISP* s in a neutral network or a non-privileged ISP_2 can not choose large *e2e QoS*, However, the CP does not have a large bandwidth to ensure *e2e QoS* promised by the privileged ISP_1, even-if it chooses *e2e QoS* as much as possible according to the c bandwidth arriving to choose $q^{\sim max}$, and for bandwidth values (between 3000 and 78000), the *ISP* s of a neutral network are increasing their *e2e QoS* in a remarkable manner with respect to the ISP_2 non-neutral network until all *ISP* s choose $q^{\sim max}$, This is the case where the bandwidth ranges from (78000 - 200000).

Figure 7 shows the demand partitioning between *ISP* s w.r.t the bandwidth c, we note that for *ISP* s in a neutral network have the same demands and proportionally increase with c until 7800, and demand is constant for all *ISP* s. For non-neutral network, and for small bandwidth values (between 1600 and 3000), *ISP* s in a neutral network or non-privileged ISP_2 can not choose great values of the *e2e QoS*, however, the CP does not have a large bandwidth to ensure promised *e2e QoS* of the privileged ISP_1, even-if this latter choose a largest *e2e QoS* possible w.r.t bandwidth, getting to choose $q^{\sim max}$, and for bandwidth values (between 3000 and 78000), the *ISP* s of a neutral network are increasing their *e2e QoS* in a remarkable manner w.r.t the ISP_2 in non-neutral network until all *ISP* s choose $q^{\sim max}$, it is the case of a bandwidth between 78000 and 200000.

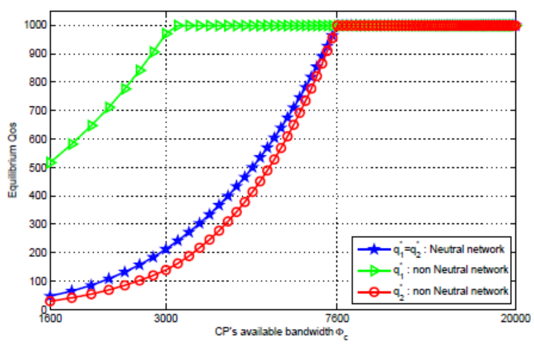

Figure 6. Impact of the available bandwidth ϕc on equilibrium QoS

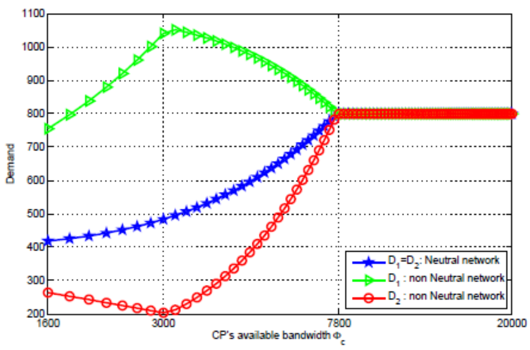

Figure 7. Impact of the available bandwidth ϕc of the CP on partitioning demand

Figure 8 represents the net revenue of the *ISP* s w.r.t the c bandwidth, in the neutral network, the *ISP* s equitably share the net revenue between them. However, the privileged ISP_1 (if the non-neutrality) attracts a large market share which impoverished the net revenue of the ISP_2. We remark that there are three regimes mainly due to the variation of the *e2e QoS*, well, usually when the *ISP* s choose $q^{\sim max}$, we note that there is a certain difference in net revenue due to the absence of penalty on quality of service, but this difference will disappear when bandwidth c tends to great values. Compared with the results of net revenue when price changes with fixed *e2e QoS*, the results of our study will allow *ISP* s to achieve significant net revenue.

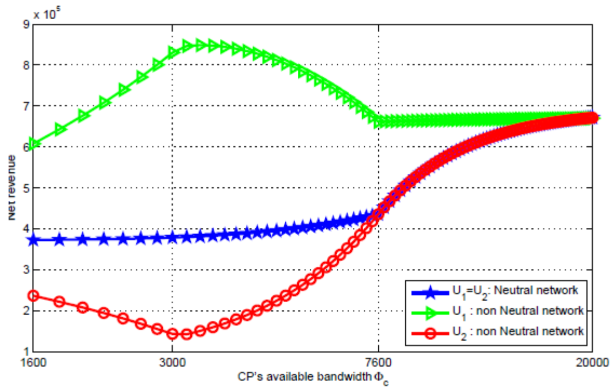

Figure 8. Impact of the available bandwidth ϕc of the CP on the net revenue

5.2.1. Neutral setup VS. non-neutral setup

To give priority to its contents over that of the ISP_2, The ISP_1 signed a special contract with the content provider. This contract can be considered as roadblocks or shortcuts defined by the *CP* to discern the content of the ISP_1. Figures 6 and 7 illustrate respectively, the change in *e2e QoS*, and the demands at the Nash equilibrium, by varying the quality of c bandwidth offered by *CP* . We note that the equilibrium *e2e QoS* for all *ISP* s in a neutral network and a non-neutral network increase with c. A special feature is that the *e2e QoS* at equilibrium of non-neutrality are more important than the *e2e QoS* in a neutral network. This encourages customers to purchase services, which explains the increase in total demand in non-neutral networks. Otherwise, the ISP_1 has more power and becomes the master of the market, giving it the ability to offer services with better quality. This causes attract new subscribers, reducing demand for the ISP_2 and even its turnover. When c is relatively low and not sufficient to answer to the total demand, the ISP_2 without privileged contract makes less than its competitor, this comes from a lack of bandwidth was consumed by the ISP_1. We note that when the content provider manages an enormous bandwidth to answer the total demand, the measures of neutrality meet the measures of non-neutrality. Thus, the ISP_1 had no reason to invest in the signing of a privileged contract since the two regimes (neutral and non-neutral) give the same result.

5.2.2. Non-neutrality sustains monopolistic and unfair competition

The figure 8 represents the net revenue of the both internet service providers for the neutral case and the non-neutral case. Due to the absence of penalty on the quality of service (*QoS* since announced is encountered in non-neutral conditions), the ISP_1 attracts a higher market share. Clearly this situation-where the ISP_1 has advantages over the ISP_2 is completely unfair. In fact, this induces a kind of monopoly position among *ISP* s. However, this monopoly situation implicitly prohibits the competitors from entering the market by using unfair competitive practices arising from its influence on the market as a privileged *ISP* .

6. IMPACT OF COMPETITION

6.1. Price of anarchy

The concept of the social surplus [12] or total cost [13], is defined as the maximum of the sum of utilities of all agents in the systems (i.e. Providers). It is well known in game theory that selfishness of the agent, as in a Nash equilibrium, typically does not lead to a socially effective situation. As a measure of efficiency loss due to divergence of interests of users, we use the price of anarchy P oA [14], this latter is a measure of the loss of efficiency due to the selfishness of the actors. This loss was defined in [14] as the ratio of the worst comparing the measure of the overall efficiency (to be selected) at the end of non-cooperative game played between the actors, to the optimum value of this measure efficiency. A P oA close to 1 indicates that the equilibrium is about socially optimal, and then the consequences of selfish behavior are relatively benign. The term price of anarchy was used by Koutsoupias and Papadimitriou [14]. As in

[15], measuring the loss of efficiency due to the selfishness of the actors as the quotient of the social welfare obtained at the Nash equilibrium and the maximum value of social welfare:

$$PoA = \frac{\min_{p,\tilde{q}} W_{NE}(p,\tilde{q})}{\max_{p,\tilde{q}} W(p,\tilde{q})} \tag{13}$$

where $W(p,\tilde{q}) = \sum_{i=1}^{N} U_i(p,\tilde{q})$ is a function of welfare and $W_{NE}(p,\tilde{q}) = \sum_{i=1}^{N} U_i(p^*,\tilde{q}^*)$ is a sum of utilities of all actors in the Nash equilibrium.

We represent the variation in the price of anarchy if the price payable by *ISP* s is fixed (Figure 9). The first remark is that the price of anarchy exceeds 0.5, it means that the equilibrium is socially acceptable for any value of bandwidth, but the price of anarchy varies according to three situations. The first when we have no enough bandwidth, the *ISP* s do not express their selfish behavior. The second situation where the *CP* has a bandwidth medium, the *ISP* s are becoming more and more selfish, and the third one where we have a great value of the bandwidth c, the selfish behavior weakens. But generally, neutrality is socially good for *ISP* s when the bandwidth c is low, by cons, when c is important, the price of anarchy of a non-neutral network is higher compared to that of the neutral network. Thus when c tends to great values, each case of neutrality become socially responsible.

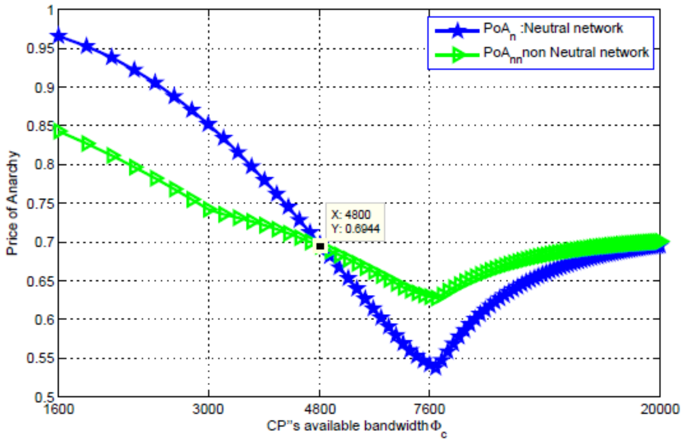

Figure 9. Impact of the available bandwidth ϕc of the CP on the price of anarchy

7. CONCLUSION

We presented in this work a non-cooperative game market share. Each *ISP* reports some *e2e QoS* reference it claims to guarantee to its subscribers. Then, each *ISP* , taking into account the demand generated, determines the best *e2e QoS* maximizes its net revenue. In addition, both neutral (no discrimination on the data flowing through the network) and non-neutral when some specific *ISP* signed a special contract with the content provider to focus its con-tent. Based on the Rosen’s Supermodularity condition, we proved the existence and uniqueness of a Nash equilibrium for both cases. We have shown numerically that the non-neutral regime is beneficial for *EU* s because it involves at great rates and improved quality of service. However, it can support the monopolistic and unfair competition between internet service providers.

APPENDICES

a. Existence of The Nash Equilibrium e2e QoS in Neutral Network

Proof 1 Equation (14) represent the second derivative of the utility function 11 w.r.t the quality of service:

$$\frac{\partial^2}{\partial^2 \tilde{q}_i} U_i(p,\tilde{q}) = -\frac{2\vartheta_i c_i}{(1-c_i\tilde{q}_i)^3} - \frac{2\theta_i \left(\beta_i^i - \sum_{k,k\neq i} \beta_k^i\right) \left(\Phi_c - D(p,\tilde{q})\right)^2 (1-c_i\tilde{q}_i)\left(\Phi_c - D(p,\tilde{q}) + \tilde{q}_i\right)}{\left(\tilde{q}_i + (1-c_i\tilde{q}_i)\left(\Phi_c - D\left(p,\tilde{q}_i\right)\right)\right)^3}$$
$$- \frac{2\theta_i \left(\left(\Phi_c - D(p,\tilde{q})\right)^2 - \left(\beta_i^i - \sum_{k,k\neq i} \beta_k^i\right) \tilde{q}_i{}^2\right)\left(-\left(\beta_i^i - \sum_{k,k\neq i} \beta_k^i\right)(1-c_i\tilde{q}_i) - \left(\Phi_c - D(p,\tilde{q})\right)c_i + 1\right)}{\left(\tilde{q}_i + (1-c_i\tilde{q}_i)\left(\Phi_c - D(p,\tilde{q})\right)\right)^3} \tag{14}$$

and we have, $1 - \left(\Phi_c - D(p,\tilde{q})\right)c_i > 0$. *So,*

$$\frac{\partial^2}{\partial^2 \tilde{q}_i} U_i(p,\tilde{q}) < -\frac{2\theta_i \left(\beta_i^i - \sum_{k,k\neq i} \beta_k^i\right)\left(\Phi_c - D(p,\tilde{q})\right)(1-c_i\tilde{q}_i)\left(\Phi_c - D(p,\tilde{q}) + \tilde{q}_i\right)\left(\Phi_c - D(p,\tilde{q}) - 1\right)}{\left(\tilde{q}_i + (1-c_i\tilde{q}_i)\left(\Phi_c - D\left(p,\tilde{q}_i\right)\right)\right)^3}$$

Then

$$\frac{\partial^2}{\partial^2 \tilde{q}_i} U_i(p,\tilde{q}) < 0 \tag{15}$$

The second derivative of the utility function is negative, then the utility function is concave, hence the existence of the Nash equilibrium e2e QoS follows, [16]

B. Uniqueness of the Nash Equilibrium e2e QoS in Neutral Network

Proof 2 The uniqueness of the equilibrium point is guaranteed if the utility function satisfies the conditions of Rosen[16], Moulin [17], derived from the supermodularity condition, which is another alternative to satisfy the conditions of Rosen: The Nash equilibrium point is unique if:

$$-\frac{\partial^2}{\partial^2 \tilde{q}_i} U_i(p,\tilde{q}) - \sum_{j,j\neq i} \left|\frac{\partial^2}{\partial \tilde{q}_i \partial \tilde{q}_j} U_i(p,\tilde{q})\right| > 0 \tag{16}$$

the mixed partial derivative is given by:

$$\frac{\partial^2}{\partial \tilde{q}_i \partial \tilde{q}_j} U_i(p,\tilde{q}) = -\frac{2\theta_i\tilde{q}_i \left(\beta_j^j - \sum_{k,k\neq j} \beta_k^j\right)\left(\Phi_c - D(p,\tilde{q}) + \tilde{q}_i(1-c_i\tilde{q}_i)\left(\beta_i^i - \sum_{k,k\neq i} \beta_k^i\right)\right)}{\left(\tilde{q}_i + (1-c_i\tilde{q}_i)\left(\Phi_c - D(p,\tilde{q})\right)\right)^3} < 0 \tag{17}$$

we pose

$$A = -\frac{\partial^2}{\partial^2 \tilde{q}_i} U_i(p,\tilde{q}) - \sum_{j,j\neq i} \left|\frac{\partial^2}{\partial \tilde{q}_i \partial \tilde{q}_j} U_i(p,\tilde{q})\right| \tag{18}$$

we verify that:

$$A > 0 \tag{19}$$

From the progression A, and after substitution of (14) and (17) in (16), we have:

$$A = \frac{2\vartheta_i c_i}{(1-c_i\tilde{q}_i)^3} + \frac{2\theta_i\left(\beta_i^i - \sum\limits_{k,k\neq i}\beta_k^i\right)(\Phi_c - D(p,\tilde{q}))^2(1-c_i\tilde{q}_i)(\Phi_c - D(p,\tilde{q})+\tilde{q}_i)}{(\tilde{q}_i + (1-c_i\tilde{q}_i)(\Phi_c - D(p,\tilde{q}_i)))^3} -$$
$$\frac{2\theta_i\left((\Phi_c - D(p,\tilde{q}))^2 - \left(\beta_i^i - \sum\limits_{k,k\neq i}\beta_k^i\right)\tilde{q}_i^2\right)\left(\left(\beta_i^i - \sum\limits_{k,k\neq i}\beta_k^i\right)(1-c_i\tilde{q}_i) + (\Phi_c - |D(p,\tilde{q}))c_i - 1\right)}{(\tilde{q}_i + (1-c_i\tilde{q}_i)(\Phi_c - D(p,\tilde{q})))^3}$$
$$-\left(\frac{2\theta_i\tilde{q}_i\left(\Phi_c - D(p,\tilde{q}) + \tilde{q}_i(1-c_i\tilde{q}_i)\left(\beta_i^i - \sum\limits_{k,k\neq i}\beta_k^i\right)\right)}{(\tilde{q}_i + (1-c_i\tilde{q}_i)(\Phi_c - D(p,\tilde{q})))^3}\right)\sum_{j,j\neq i}\left(\beta_j^j - \sum_{k,k\neq j}\beta_k^j\right)$$

on the assumption 3, we have: $\beta_i^i - \sum\limits_{j,j\neq i}\beta_i^j \leqslant \frac{1}{N}$, $\forall i,j = 1,...,N.$*, then we have:*

$$A > \frac{2\vartheta_i c_i}{(1-c_i\tilde{q}_i)^3} + \frac{2\theta_i(\Phi_c - D(p,\tilde{q})+\tilde{q}_i)\left((\Phi_c - D(p,\tilde{q}))(\Phi_c - D(p,\tilde{q}))\left(\frac{(1-c_i\tilde{q}_i)}{N}\right) - \tilde{q}_i\right)}{(\tilde{q}_i + (1-c_i\tilde{q}_i)(\Phi_c - D(p,\tilde{q}_i)))^3}$$
$$-\frac{2\theta_i(\Phi_c - D(p,\tilde{q})+\tilde{q}_i)(\Phi_c - D(p,\tilde{q}) - \tilde{q}_i)\left(\frac{(1-c_i\tilde{q}_i)}{N}\right)}{(\tilde{q}_i + (1-c_i\tilde{q}_i)(\Phi_c - D(p,\tilde{q})))^3}$$

with

$$\Phi_c - D(p,\tilde{q}) = \frac{\tilde{q}_i}{(1-c_i\tilde{q}_i)} + \sum_{k,k\neq i}\frac{\tilde{q}_k}{(1-c_k\tilde{q}_k)}$$

then,

$$A > \frac{2\vartheta_i c_i}{(1-c_i\tilde{q}_i)^3} + \frac{2\theta_i\tilde{q}_i(\Phi_c - D(p,\tilde{q})+\tilde{q}_i)\left(\frac{\Phi_c - D(p,\tilde{q})}{N} - 1\right)}{(\tilde{q}_i + (1-c_i\tilde{q}_i)(\Phi_c - D(p,\tilde{q}_i)))^3}$$
$$+\frac{2\theta_i(\Phi_c - D(p,\tilde{q})+\tilde{q}_i)(1-c_i\tilde{q}_i)\left(\frac{\Phi_c - D(p,\tilde{q})}{N}\right)\left(\sum\limits_{k,k\neq i}\frac{\tilde{q}_k}{(1-c_k\tilde{q}_k)} - 1\right)}{(\tilde{q}_i + (1-c_i\tilde{q}_i)(\Phi_c - D(p,\tilde{q})))^3}$$

For $\tilde{q}_i \geqslant \frac{1}{1+c_i} \simeq 1$ *we have:* $\frac{\tilde{q}_i}{(1-c_i\tilde{q}_i)} \geqslant 1$ *which implies that* $\Phi_c - D(p,\tilde{q}) = \sum\limits_k \frac{\tilde{q}_i}{(1-c_i\tilde{q}_i)} \geqslant N$
Finally we have:

$$A = -\frac{\partial^2}{\partial^2\tilde{q}_i}U_i(p,\tilde{q}) - \sum_{j,j\neq i}\left|\frac{\partial^2}{\partial\tilde{q}_i\partial\tilde{q}_j}U_i(p,\tilde{q})\right| > 0 \qquad (20)$$

Regarding the assumption 1 and according to the remark 1, gives the positivity of the last expression. This means that the supermodularity condition of Moulin holds, then this game satisfies the conditions of Rosen, where the uniqueness of the equilibrium is verified following, [17].

C. Existence and Uniqueness of the Nash Equilibrium e2e QoS in Non Neutral Net-Work

Proof 3 It is clear that the expression of the utility function of ISP s who have no privileged contract with the CP , is the same as that of ISP s in the case of net neutrality, namely, equation (11). Now the utility function of these ISP s, satisfies the properties of the concavity and uniqueness, equation (16). Therefore, to prove the existence and uniqueness of equilibrium under conditions of non-neutrality, it suffices to prove that the utility function of the ISP1 (12)), verifies the concavity properties (existence) and the state of supermodularity, equation (16), (Uniqueness).

C1. Existence

Equation (21) represents the second derivative of the utility function of ISP_1:

$$\frac{\partial^2}{\partial^2 \tilde{q}_1} U_1(p, \tilde{q}) = -\frac{2\vartheta_1 \left(\Phi_c - D_1(p, \tilde{q})) - {\beta_1^1}^2 \tilde{q}_1\right)}{(\Phi_c - D_1(p, \tilde{q})) - \tilde{q}_1)^2} - \frac{2\vartheta_1 \tilde{q}_1 \left(\beta_1^1 (\Phi_c - D_1(p, \tilde{q}) + 1)\right)^2}{(\Phi_c - D_1(p, \tilde{q})) - \tilde{q}_1)^3} \quad (21)$$

Since we have $\tilde{q}_1 = \frac{1}{Delay_u^1 + c_1}$, *then* $1 - \tilde{q}_1 c_1 \geq 0$, *and,* $c_1 = \frac{1}{\Phi_c - D_1(p, \tilde{q})}$, *we notice that,* $\Phi_c - D_1(p, \tilde{q}) - \tilde{q}_1 \geq 0$. *Then it is easy to note (Equation (21)) that:*

$$\frac{\partial^2}{\partial^2 \tilde{q}_1} U_1(p, \tilde{q}) < 0$$

Thus, the utility function is concave, thereafter, the existence of a Nash equilibrium is the result of [16].

C2. Uniqueness

$$\frac{\partial^2}{\partial \tilde{q}_1 \partial \tilde{q}_l} U_1(p, \tilde{q}) = \frac{2\vartheta_1 \tilde{q}_1 \beta_1^l \left(\Phi_c - D_1(p, \tilde{q}) + \tilde{q}_1 \beta_1^1\right)}{(\Phi_c - D_1(p, \tilde{q}) - \tilde{q}_1)^3} > 0 \quad (22)$$

Assume that:

$$B = -\frac{\partial^2}{\partial^2 \tilde{q}_1} U_1(p, \tilde{q}) - \sum_{l, l \neq 1} \left| \frac{\partial^2}{\partial \tilde{q}_1 \partial \tilde{q}_l} U_1(p, \tilde{q}) \right|$$

we verify that:

$$B > 0$$

then

$$\begin{aligned} B &= \frac{2\vartheta_1 \left(\Phi_c - D_1(p, \tilde{q})) - {\beta_1^1}^2 \tilde{q}_1\right)}{(\Phi_c - D_1(p, \tilde{q})) - \tilde{q}_1)^2} + \frac{2\vartheta_1 \tilde{q}_1 \left(\beta_1^1 (\Phi_c - D_1(p, \tilde{q}) + 1)\right)^2}{(\Phi_c - D_1(p, \tilde{q})) - \tilde{q}_1)^3} - \sum_{l, l \neq 1} \left(\frac{2\vartheta_1 \tilde{q}_1 \beta_1^l \left(\Phi_c - D_1(p, \tilde{q}) + \tilde{q}_1 \beta_1^1\right)}{(\Phi_c - D_1(p, \tilde{q}) - \tilde{q}_1)^3} \right) \\ &> \frac{2\vartheta_1}{(\Phi_c - D_1(p, \tilde{q})) - \tilde{q}_1)^3} \left[(\Phi_c - D_1(p, \tilde{q}))) \left(\Phi_c - D_1(p, \tilde{q})) + (1 - {\beta_1^1}^2) \tilde{q}_1\right) + {\tilde{q}_1}^2 \beta_1^1 \left(\beta_1^1 - \sum_{l, l \neq 1} \beta_1^l\right) \right] \\ &+ \frac{2\vartheta_1 \tilde{q}_1 \beta_1^1 (\Phi_c - D_1(p, \tilde{q}))}{(\Phi_c - D_1(p, \tilde{q})) - \tilde{q}_1)^3} \left[\beta_1^1 - \sum_{l, l \neq 1} \beta_1^l \right] > 0 \end{aligned}$$

So

$$-\frac{\partial^2}{\partial^2 \tilde{q}_1} U_1(p, \tilde{q}) - \sum_{l, l \neq 1} \left| \frac{\partial^2}{\partial \tilde{q}_1 \partial \tilde{q}_l} U_1(p, \tilde{q}) \right| > 0. \quad (23)$$

This result shows that for the ISP1 utility function, the supermodularity condition of Moulin is satisfied, hence the uniqueness of equilibrium.

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