# Analysis of Competition Between Content Providers in the Internet Market

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Article Info	ABSTRACT			
Article history:	In the Internet market, content providers (CPs) continue to play a primordial role in the pro-cess of accessing different types of data: Images, Texts, Videosetc. Competition in this area is fierce, customers are looking for providers that offer them good content ( credibility of content and quality of service) with a reasonable price. In this work, we analyze this competition between CPs and the economic influence of their strategies on the market. We formulate our problem as a non-cooperative game among multiple CPs			
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Keyword:	for the same market. Through a detailed analysis, we prove uniqueness of pure Nash Equilibrium (NE). Further-more, a fully distributed algorithm to			
Content Providers	converge to the NE point is presented. In order to quantify how efficient is			
Game Theory	the NE point, a detailed analysis of the Price of Anarchy (PoA) is adopted to ensure the performance of the system at equilibrium. Finally, we provide an			
Nash Equilibrium Price of Anarchy Pricing-QoS	extensive numerical study to describe the interactions between CPs and to point out the importance of quality of service (QoS) and credibility of content in the market.			
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# 1. INTRODUCTION

The current internet has enabled numerous distributed applications and services. However, providers gener-ally face many challenges in determining technical and economic solutions to providing services[1]. Key challenges are how to price and bill these services and how to establish economic relationships with other providers that are necessary to provide end-to-end services. Equilibrium models for the internet generally assume basic economic relationships and consider price as the only factor that affects users demand (see [2][3][4]). However, in new paradigms for the internet and even in the case of supply chain networks, price is not the only factor and Quality of Service (QoS), i.e., the ability to provide different priorities to applications, users, or data flows, comes into play (see [5] [6] [7] [8] [9] ).

Our contribution in this work is to expand the study on the internet domain by adding a utility model on income from content providers (CPs). CPs can be social networks, internet search engines, or any other websites. In this framework, we propose a modeling competition between CPs based on the parameters of price and credibility of content that is used to measure the effectiveness of content provided by the CP. Credibility is a function that depends on the QoS and quality of content (QoC).

Customer behavior is modeled by the function demand that depends on providers policies. We use game theory to study the behavior of CPs in the internet. Then we study the impact of their decisions on customers and other CPs. We focus our studies on the non-cooperative games in terms of stable solutions, which are the pure strategy Nash Equilibria (NE) of the game. We do not consider mixed strategy equilibria, because our environment requires a concrete strategy rather than a randomized strategy, which would be the

result of a mixed strategy. Hence, when using the term Nash equilibrium we mean pure strategy exact NE unless mentioned otherwise.

CPs may be faced with the question of "how to choose in what content to specialize" [10]. E. Altman considers several CPs that are faced with a similar problem and study the impact of their decisions on each other using a game theoretic approach. The author shows that the problem of selecting the content type is equivalent to a congestion game. In [11], the authors have studied game problems involving two types of CPs : one that corresponds to independent CPs, and one that correspond to CPs that have exclusive agreements with Internet Service Providers (ISPs). The cost for the internet users who are subscribers of some ISP of fetching content from an independent CP or from a CP that has an exclusive agreement with another ISP, was assumed to be larger than for fetching it from the CP that has an exclusive agreement with their own ISP.

Weijie Wu et al. consider a Stackelberg game, where the CP decides reward first, and after that, the peers decide amount of capacity [5]. The CP rewards the peers based on the amount of upload capacity they contribute. From CP point of view, it aims at minimizing its total cost, i.e., the cost of uploading and the cost of rewarding the peers. The utility of a peer is the reward it receives, minus its cost of upload contribution.

The rest of the paper is organized as follows : in section 2., we describe the problem model. In section 3., we formulate our model as a non-cooperative game. We present numerical results in section 4. and conclude our work in section 5.

### 2. PROBLEM MODELING

In this section, we formulate the interaction among content providers (CPs) as a non-cooperative game. Each CP chooses credibility of content and the corresponding price. We consider a system with N content providers. Let pi and ci, be, respectively, the tariff and the credibility of content guaranteed by CPi. Now, each customer seeks to the content provider which allows him to meet a credibility of content sufficient to satisfy his/her needs, at suitable price. We consider that behaviors of customer's has been handled by a simple function so called demand functions, see equation (1). This later depends on the price and credibility of content strategies of all. CPs are supposed to know the effect of their policy on the customer's.

#### 2.1. Model of Content Credibility Function

We assume that the function of the credibility of content ci of CPi is a function of the quality of service qsi and quality content qci above which is written as follows :

$$c_i = \lambda q_{s_i} + \mu q_{c_i} \tag{1}$$

 $\lambda$  and  $\mu$  are two positive constants, which represent respectively the sensitivity of the credibility of content to QoS and QoC, such as :

$$\lambda + \mu = 1$$
 (2)

That the QoS as the "expected delay", which is computed by the Kleinrock function (see [12]) as the reciprocal of the square root of delay :

$$q_{s_i} = \frac{1}{\sqrt{Delay}} = \sqrt{b_i - D_i} \tag{3}$$

 $b_i$  is the amount of bandwidth required by  $CP_i$ .

The quality of content provided can be specified for a specific domain of content, e.g., video streaming.

### 2.2. Demand Model

we consider that the demand function  $D_i$  of the  $CP_i$  is linear with respect to the set price  $p_i$  and the credibility  $c_i$ , see [13]. This demand function depends also on prices  $p_{-i}$  and  $c_{-i}$  set by the competitors. Namely, the demand function of  $CP_i$  depends on  $p=[p_1,..,p_N]$  and  $c=[c_1,..,c_N]$ . Eventually,  $D_i$  is decreasing w.r.t.  $p_i$  and increasing w.r.t.  $p_j$ ,  $j \neq i$ . Whereas it is increasing w.r.t  $c_i$  and increasing w.r.t.  $c_j$ ,  $j \neq i$ . Then, the demand functions w.r.t services of  $CP_i$  can be written as follows:

$$D_i(p,c) = D_0^i - \alpha_i^i p_i + \beta_i^i c_i + \sum_{j,j \neq i} \left[ \alpha_i^j p_j - \beta_i^j c_j \right]$$

$$\tag{4}$$

where  $D_0^i$  is a positive constant used to insure non-negative demands over the feasible region. While  $\alpha_i^j$  and  $\beta_i^j$  are positive constants representing respectively the sensitivity of  $CP_i$  to price and credibility of content of  $CP_j$ .

$$\sum_{j=1}^{N} \alpha_i^j = 1 \ et \ \sum_{j=1}^{N} \beta_i^j = 1 \ , \ i = 1, ..., N.$$
(5)

Assumption 1 For any pricing profile, the price mutual sensitivities satisfy :

$$\alpha_{i}^{i} \geqslant \sum_{j,j \neq i} \alpha_{i}^{j} \hspace{0.2cm}, \hspace{0.2cm} \forall \hspace{0.1cm} i,j = 1,...,N$$

Assumption 2 For any credibility of content profile, the credibility mutual sensitivities satisfy :

$$\beta_i^i \geqslant \sum_{j,j \neq i} \beta_i^j \ , \ \forall \ i,j=1,...,N.$$

Assumption 1 and 2 will be needed to ensure the uniqueness of the resulting equilibrium. It is furthermore a reasonable condition, in that Assumption 1 (resp. Assumption 2) implies that the influence of a CP price (resp. credibility of content) is significantly greater on its observed demand than the prices (resp. credibilities of content) of its competi-tors. This condition could then take into account the presence of customer loyalties and/or imperfect knowledge of competitors prices [13].

# 2.3. Utility model for CP

$$U_i(p, q_s, q_c) = p_i D_i(p, q_s, q_c) - F_i(p, q_s, q_c), \forall i \in \{1, ..., N\}.$$
(6)  
where  $F_i(p, q_s, q_c)$  the fee paid by  $CP_i$  (investment of  $CP_i$ ):

$$F_i(p, q_s, q_c) = C_s^i + H_s^i$$
(7)

.  $C_s^i$  the total cost of content provider  $CP_i$  to produce the credibility of content  $c_i$ :

$$C_{s}^{i} = \theta_{i}c_{i}$$
. (8)

where  $\theta_i$  is the cost per unit of credibility of content for  $CP_i$ .

.  $H_s^i$  is the total cost to transmit the contents with credibility  $c_i$ , i.e.:

$$H_s^i = \gamma_i D_i(p, q_s, q_c)c_i \tag{9}$$

where  $\gamma_i$  the cost to transmit a content unit with credibility  $c_i$ .

The equation 7 becomes :

$$F_i(p, q_s, q_c) = \theta_i c_i + \gamma_i D_i(p, q_s, q_c) c_i$$
(10)

Then, starting from the equations 2,8 and 9 the function of utility of the  $CP_i$  can be written in the following way :

$$U_i(p, q_s, q_c) = p_i D_i(p, q_s, q_c) - \theta_i (\lambda q_{s_t} + \mu q_{c_t}) - \gamma_i D_i(p, q_s, q_c) (\lambda q_{s_t} + \mu q_{c_t})$$
(11)

#### 3. NON-COOPERATIVE GAME FORMULATION

Let  $G = [\mathcal{N}, \{P_i, Q_{s_i}, Q_{c_i}\}, \{U_i(.)\}]$  denote the non-cooperative game, where  $\mathcal{N} = \{1, ..., N\}$  is the index set identifying the CPs,  $P_i$  is the price strategy set of  $CP_i$ ,  $Q_{s_i}$  is the QoS strategy set of  $CP_i$ ,  $Q_{c_i}$  is the QoC strategy set of  $CP_i$  and  $U_i(.)$  is the utility function. Let the price vector  $p = (p_1, ..., p_N)^T \in P^N = P_1 \times P_2 \times ... \times P_N$ , QoS vector  $q_s = (q_{s_i}, ..., q_{s_N})^T \in Q_s^N = Q_{s_1} \times Q_{s_2} \times ... \times Q_{s_N}$ , QoC vector  $q_c = (q_{c_i}, ..., q_{c_N})^T \in Q_c^N = Q_{c_1} \times Q_{c_2} \times ... \times Q_{c_N}$ . The utility of  $CP_i$  when it decides the strategy  $p_i$ ,  $q_{s_i}$  et  $q_{c_i}$  is given in equation 11. We assume that the strategy spaces  $P_i$ ,  $Q_{s_i}$  and  $Q_{c_i}$  of each CP are compact and convex sets with maximum and minimum constraints, et For any given  $CP_i$  we consider strategy spaces the closed intervals  $P_i = [\underline{p_i}, \overline{p_i}], Q_{s_i} = [\underline{q_{s_i}}, \overline{q_{s_i}}]$  et

$$Q_{c_i} = \left| \underline{q_{c_i}}, \overline{q_{c_i}} \right|.$$

In order to maximize their utilities, each  $CP_i$  decides a price  $p_i$ , QoS  $q_{s_i}$  and QoC  $q_{c_i}$ . Formally, the problem can be expressed as :

$$\max_{p_i \in P_i, q_{s_i} \in Q_{s_i}, q_{c_i} \in Q_{c_i}} U_i(p, q_s, q_c), \quad \forall i \in \mathcal{N}.$$

#### 3.1. QoS Game

Considering some fixed price and QoC policy, a Nash equilibrium in QoS is formally defined as :

**Definition 1** A QoS vector  $q_s^* = (q_{s_1}^*, ..., q_{s_N}^*)$  is a Nash equilibrium of the game  $G = [\mathcal{N}, \{P_i, Q_{s_i}, Q_{c_i}\}, \{U_i(p, q_s, q_c)\}]$  if :

$$\forall (i, q_{s_i}) \in (\mathcal{N}, Q_{s_i}), \ U_i(q_{s_i}^*, q_{s_{-i}}^*) \ge U_i(q_{s_i}, q_{s_{-i}}^*)$$

**Theorem 1** A Nash equilibrium in terms of QoS for game  $G = [\mathcal{N}, \{P_i, Q_{s_i}, Q_{c_i}\}, \{U_i(p, q_s, q_c)\}]$  exists and is unique.

**Proof :** To prove existence, we note that each CPs strategy space  $Q_{s_i}$  is defined by all QoSs in the closed interval bounded by the minimum and maximum QoSs. Thus, the joint strategy space  $Q_s$  a nonempty, convex, and compact subset of the Euclidean space  $R^N$ . In addition, the utility functions are concave with respect to QoSs as can be seen from the second derivative test:

$$\frac{\partial^2}{\partial q_{s_i}^2} U_i(p, q_s, q_c) = -2\gamma_i \beta_i^i \lambda^2 \le 0, \quad \forall i \in \mathcal{N}$$
(12)

which ensures existence of a Nash equilibrium.

Uniqueness of the equilibrium point is guaranteed if the utility function satisfies Rosen's conditions [14]. In [15], Moulin derived the dominance solvability condition, which is another alternative to satisfy Rosen's conditions : The Nash equilibrium point is unique if :

$$-\frac{\partial^2}{\partial q_{s_i}^2} U_i(p, q_s, q_c) - \sum_{j, j \neq i} \left| \frac{\partial^2}{\partial q_{s_i} \partial q_{s_j}} U_i(p, q_s, q_c) \right| \ge 0$$
(13)

$$\frac{\partial^2}{\partial q_{s_i} \partial q_{s_j}} U_i(p, q_s, q_c) = \gamma_i \beta_i^j \lambda^2.$$
(14)

After substitution of 12 and 14 in 13, we have :

$$-\frac{\partial^2}{\partial q_{s_i}^2} U_i(p, q_s, q_c) - \sum_{j, j \neq i} \left| \frac{\partial^2}{\partial q_{s_i} \partial q_{s_j}} U_i(p, q_s, q_c) \right| = \gamma_i \lambda^2 \left( 2\beta_i^i - \sum_{j, j \neq i} \beta_i^j \right) \ge 0.$$
(15)

The fixed price QoC Nash equilibrium point is then unique and is given by :

$$q_{s_i}^* \in \underset{q_{s_i} \in Q_{s_i}}{\operatorname{argmax}} U_i(q_{s_i}, q_{s_{-i}}^*), \quad \forall i \in \mathscr{N}.$$

$$(16)$$

#### 3.2. Price Game

Considering some fixed QoS and QoC policy, a Nash equilibrium in price is formally defined as :

**Definition 2** A price vector  $p^* = (p_1^*, ..., p_N^*)$  is a Nash equilibrium of the game  $G = [\mathcal{N}, \{P_i, Q_{s_i}, Q_{c_i}\}, \{U_i(p, q_s, q_c)\}]$  if :

$$\forall (i, p_i) \in (\mathcal{N}, P_i), \ U_i(p_i^*, p_{-i}^*) \ge U_i(p_i, p_{-i}^*)$$

**Theorem 2** A Nash equilibrium in terms of price for game  $G = [\mathcal{N}, \{P_i, Q_{s_i}, Q_{c_i}\}, \{U_i(p, q_{|s|}, q_c)\}]$  exists and is unique.

**Proof :** To prove existence, we note that each CPs strategy space  $P_i$  is defined by all QoSs in the closed interval bounded by the minimum and maximum prices. Thus, the joint strategy space P a nonempty, convex, and compact subset of the Euclidean space  $R^N$ . In addition, the utility functions are concave with respect to QoSs as can be seen from the second derivative test: With the same reasoning as in the QoS game subsection, the utility functions are concave with respect to prices as can be seen from the second derivative test:

$$\frac{\partial^2}{\partial p_i^2} U_i(p, q_s, q_c) = -2\alpha_i^i \le 0.$$
<sup>(17)</sup>

which ensures existence of a Nash equilibrium. The Nash equilibrium point is unique if :

$$-\frac{\partial^2}{\partial p_i^2} U_i(p, q_s, q_c) - \sum_{j, j \neq i} \left| \frac{\partial^2}{\partial p_i \partial p_j} U_i(p, q_s, q_c) \right| \ge 0$$
(18)

The mixed partial is written as :

$$\frac{\partial^2}{\partial p_i \partial p_j} U_i(p, q_s, q_c) = \alpha_i^j \tag{19}$$

After substitution of 17 and 19 in 18, we have :

$$-\frac{\partial^2}{\partial p_i^2} U_i(p, q_s, q_c) - \sum_{j, j \neq i} \left| \frac{\partial^2}{\partial p_i \partial p_j} U_i(p, q_s, q_c) \right| = 2\alpha_i^i - \sum_{j, j \neq i} \alpha_i^j \ge 0$$
(20)

The fixed QoC QoS Nash equilibrium point is then unique and is given by :

$$p_i^* \in \operatorname*{argmax}_{p_i \in P_i} U_i(p_i, p_{-i}^*), \quad \forall i \in \mathscr{N}.$$

$$(21)$$

#### 3.3. OoC Game

**Definition 3** A QoC vector  $q_c^* = (q_{c_1}^*, ..., q_{c_N}^*)$  is a Nash equilibrium of the game  $G = [\mathcal{N}, \{P_i, Q_{s_i}, Q_{c_i}\}, \{U_i(p, q_s, q_c)\}]$  if :

$$\forall (i, q_{c_i}) \in (\mathcal{N}, Q_{c_i}), \ U_i(q_{c_i}^*, q_{c_{-i}}^*) \ge U_i(q_{c_i}, q_{c_{-i}}^*)$$

**Theorem 3** A Nash equilibrium in terms of QoC for game  $G = [\mathcal{N}, \{P_i, Q_{s_i}, Q_{c_i}\}, \{U_i(p, q_s, q_c)\}]$  exists and is unique.

**Proof :** With the same reasoning as in the QoS game subsection, the utility functions are concave with respect to QoCs as can be seen from the second derivative test:

$$\frac{\partial^2}{\partial q_{c_i}^2} U_i(p, q_s, q_c) = -2\gamma_i \beta_i^i \mu^2 \le 0, \quad \forall i \in \mathcal{N}$$
(22)

The Nash equilibrium point is unique if :

$$-\frac{\partial^2}{\partial q_{c_i}^2} U_i(p, q_s, q_c) - \sum_{j, j \neq i} \left| \frac{\partial^2}{\partial q_{c_i} \partial q_{c_j}} U_i(p, q_s, q_c) \right| \ge 0$$
(23)

The mixed partial is written as :

$$\frac{\partial^2}{\partial q_{c_i}\partial q_{c_j}}U_i(p,q_s,q_c) = \gamma_i\beta_i^j\mu^2 \tag{24}$$

$$-\frac{\partial^2}{\partial q_{c_i}^2} U_i(p, q_s, q_c) - \sum_{j, j \neq i} \left| \frac{\partial^2}{\partial q_{c_i} \partial q_{c_j}} U_i(p, q_s, q_c) \right| = \gamma_i \mu^2 \left( 2\beta_i^i - \sum_{j, j \neq i} \beta_i^j \right) \ge 0$$
(25)

The fixed price QoS Nash equilibrium point is then unique and is given by :

$$q_{c_i}^* \in \operatorname*{argmax}_{q_{c_i} \in Q_{c_i}} U_i(q_{c_i}, q_{c_{-i}}^*), \quad \forall i \in \mathcal{N}.$$

$$\tag{26}$$

## 3.4. Joint Price, QoC and QoS Game

In the subsections cited above, we have shown the existence and the uniqueness of NE, by fixing each time one of the parameters. The next task is to determine the price, QoS and QoC at equilibrium. This calculation will be based on the best response algorithm:

### Algorithm 1 Best Response Algorithm

- 1: Initialize vectors X randomly;
- 2: For each  $CP_i$   $i \in \mathcal{N}$  at iteration t:

a)  $x_i^{t+1} = \underset{x_i \in X_i}{\operatorname{argmax}} (U_i(X)).$ 

where: X denotes p,  $q_s$  and  $q_c$  vectors and  $X_i$  denotes the price , QoS and QoC policy profiles of  $CP_i$ .

#### 3.5. Price of Anarchy

The concept of social welfare [16], is defined as the sum of the utilities of all agents in the systems (i.e. Providers). It is well known in game theory that agent selfishness, such as in a Nash equilibrium, does not lead in general to a socially efficient situation. As a measure of the loss of efficiency due to the divergence of user interests, we use the Price of Anarchy (PoA) [17], this latter is a measure of the loss of efficiency due to actors' selfishness. This loss has been defined in [17] as the worst-case ratio comparing the global efficiency measure (that has to be chosen) at an outcome of the noncooperative game played among actors, to the optimal value of that efficiency measure. A PoA close to 1 indicates that the equilibrium is approximately socially optimal, and thus the consequences of selfish behavior are relatively benign. The term Price of Anarchy was first used by Koutsoupias and Papadimitriou [17] but the idea of measuring inefficiency of equilibrium is older. The concept in its current form was designed to be the analogue of the "approximation ratio" in Approximation Algorithms or the "competitive ratio" in Online Algorithms. As in [18], we measure the loss of efficiency due to actors selfishness as the quotient between the social welfare obtained at the Nash equilibrium and the maximum value of the social welfare:

$$PoA = \frac{\min_{p,q_s,q_c} W_{NE}(p,q_s,q_c)}{\max_{p,q_s,q_c} W(p,q_s,q_c)}$$
(27)

where  $W(p, q_s, q_c) = \sum_{i=1}^{N} U_i(p, q_s, q_c)$  is a welfare function and  $W_{NE}(p^*, q_s^*, q_c^*) = \sum_{i=1}^{N} U_i(p^*, q_s^*, q_c^*)$  is a sum of utilities of all actors at Nash Equilibrium.

# 4. NUMERICAL INVESTIGATIONS

To clarify and show how to take advantage from our theoretical study, we suggest to study numerically the market share game while considering the best response dynamics and expressions of demand as well as utility functions of CPs. Hence, we consider a system with tree CPs seeking to maximize their respective revenues. Table 1 represents the system parameter values considered in this numerical study.

rable 1. System parameters used for numerical examples				
$\frac{\alpha_1^1 = \alpha_2^2 = \alpha_3^3}{0.7}$	$\frac{\alpha_i^j, \ i \neq j}{0.15}$	$\beta_{\underline{1}}^1 = \beta_{\underline{2}}^2 = \beta_{\underline{3}}^3 \\ 0.7$	$\frac{\beta_i^j, \ i \neq j}{0.3}$	
$\lambda = \mu \\ 0.5$	$\frac{\theta_1 = \theta_2 = \theta_3}{2}$	$\frac{\gamma_1 = \gamma_2 = \gamma_3}{1}$	$\frac{\overline{D_0^1}}{400}$	
$\frac{\overline{p_1} = \overline{p_2} = \overline{p_3}}{1000}$	$\underline{p_1 = p_2 = p_3}_1$	$\overline{q_{s_1}} = \overline{q_{s_2}} = \overline{q_{s_3}}$ $1000$	$\underline{q_{s_1} = q_{s_2}} = \underline{q_{s_3}}$	
$\frac{\overline{q_{c_1}} = \overline{q_{c_2}} = \overline{q_{c_3}}}{1000}$	$\underline{q_{c_1}} = \underline{q_{c_2}} = \underline{q_{c_3}}$	$\frac{D_0^2}{300}$	$\frac{D_0^3}{200}$	

Table 1. System parameters used for numerical examples

Figures 1, 2 and 3 present respectively curves of the convergence to Nash Equilibrium Price, to Nash Equi-librium QoC and to Nash Equilibrium QoS. It is clear that the best response dynamics converges to the unique Nash equilibrium price, QoC and QoS. We also remark that the speed of convergence is relatively high (around 5 rounds are enough to converge to the joint price and QoS equilibrium).

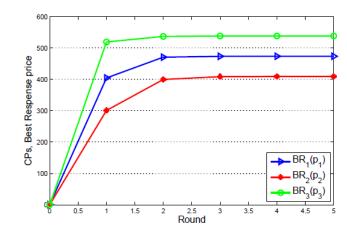


Figure 1. Price game : Convergence to the Price Nash Equilibrium

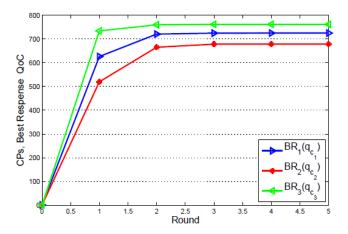


Figure 2. QoC game : Convergence to the QoC Nash Equilibrium.

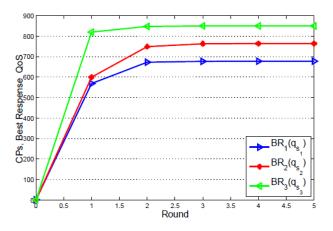


Figure 3. QoS game : Convergence to the QoS Nash Equilibrium

In the following, we discuss the impact of the system parameters on the system efficiency in terms of Price of Anarchy (PoA). Figure 4 and 5 plot the variation curve of PoA with respect to and which represents the sensitivity of CPi to his price pi and his credibility ci. In these figures, we first notice that the PoA increases when and increases, the fact that the PoA increases with and finds the simple intuition that increasing the sensitivity of CPs to their prices and their credibilities gives more and more freedom to CPs

for optimizing the Nash equilibrium. When = 11 = 22 = 33 = 1 and = 11 = 22 = 33 = 1, in the other word, when the sensitivity of an *CP* to the prices of its competitors is zero, PoA converges to 1 and the equilibrium is approximately socially optimal.

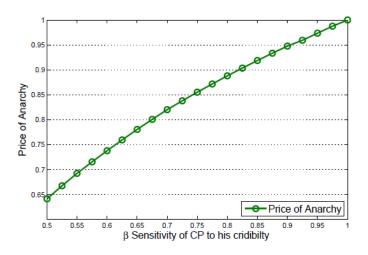


Figure 4. Price of Anarchy as a function of  $\beta = \beta_1^1 = \beta_2^2 = \beta_3^3$ 

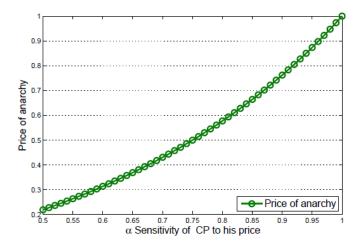


Figure 5. Price of Anarchy as a function of  $\alpha = \alpha_1^1 = \alpha_2^2 = \alpha_3^3$ 

Figure 6 shows the PoA variation curve as a function of  $\theta$ . Without loss of generality, we assume that  $\theta = \theta_1 = \theta_2 = \theta_3$ . A special feature is that the Nash equilibrium performs well and the loss of efficiency is only around 8%. This result indicates that the Nash equilibrium of this game is fair and socially efficient.

Figure 7 shows the PoA variation curve as a function of  $\gamma = \gamma_1 = \gamma_2 = \gamma_3$  which represents the cost to transmit a content unit with some credibility. In that figure, we first notice that the price of anarchy increases when  $\gamma$  increases. If the price is cheaper CPs are selfish, each CP seeks to maximize returned individually. On the other hand if the price is very expensive CPs cooperate with each other for optimizing the Nash equilibrium. Finally,  $\gamma$  control the selfishness of CPs.

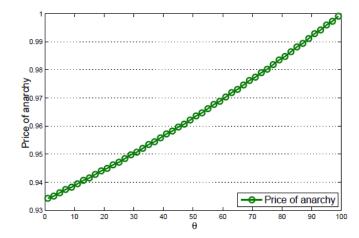


Figure 6. Price of Anarchy as a function of  $\theta = \theta_1 = \theta_2 = \theta_3$ .

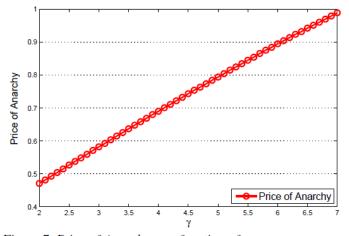


Figure 7. Price of Anarchy as a function of  $\gamma = \gamma_1 = \gamma_2 = \gamma_3$ .

# 5. CONCLUSION AND PERSPECTIVES

In this work, we presented and analyzed a framework to model the complex interactions among content providers as players through a class of two parameter Nash equilibrium model. The model is based on a simple linear demand functions which describe the behaviour of customers. These functions take into account not only the characteristics of a current but also of all other CPs in presence of two parameters describing each CP service price and credibility of content. We established uniqueness of a Nash equilibrium point and showed it with relevant numerical results. To quantify how efficient is the NE point we used PoA measurement and showed its variation according to the parameters of the model. Our proposed algorithm finds very fast the equilibrium price and the content credibility to be chosen by each CP.

This work can be developed to achieve the following purposes :

- a. Modeling the users choice with a nonlinear function which is based on the Logit model.
- b. Introducing advertising revenues in the utility of CPs to achieve maximum revenue. In 2010, advertising rev-enues of Google are more than 20 billions.
- c. Taking into consideration the purchasing power of a target market for CPs.

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