International Journal of Informatics and Communication Technology (IJ-ICT)
Vol. 13, No. 2, August 2024, pp. 143~151
ISSN: 2252-8776, DOI: 10.11591/ijict.v13i2.pp143-151

Quasi linear and stone geary utility functions based-internet service financing scheme with marginal costs and monitoring costs

Indrawati, Fitri Maya Puspita, Evi Yuliza, Oki Dwipurwani, Sisca Octarina, Rizky Helmayanti
Department of Mathematics, Faculty of Mathematics and Natural Science, Sriwijaya University, Palembang, Indonesia

Article Info

Article history:

Received Sep 19, 2023
Revised Apr 16, 2024
Accepted Apr 30, 2024

Keywords:

Consumer interest
ISP
Marginal costs
Monitoring costs
Payment options
Quasi-linier utility function
Stone-geary

ABSTRACT

The use of computer network technology is currently increasing, especially on the internet network. To connect to the actual internet, it is a task for internet service provider (ISP). Providing advantages to ISPs, it requires a financing scheme. This study's goal is to present a modified model for internet service financing schemes, within the customer choices and consumer satisfaction levels to maintain the schemes. To achieve the best outcomes, this updated model is built through marginal costs and cost monitoring while taking into account service quality based on stone-geary utility functions and quasi-linear utility functions. This research provides a solution regarding the differences in increasing consumer interest with payment options on model modification that will be provided. Traffic Digilib in a local server in Palembang. According to this study, a usage-based financing strategy and a two-part pricing of IDR 2727.8 per kbps will yield the highest revenues.

Corresponding Author:

Fitri Maya Puspita
Department of Mathematics, Faculty of Mathematics and Natural Science, Sriwijaya University
Palembang, South Sumatra 30662, Indonesia
Email: fitrimayapuspita@unsri.ac.id

1. INTRODUCTION

The use of computer network technology is currently increasing, especially in internet networks, including for internet of things (IoT) [1], [2]. To connect to our internet network we must subscribe to an internet service provider (ISP) [3]. ISP is the company that operates in the internet services sector [2]. In providing benefits, for ISPs, need a financing scheme where the scheme provided can guarantee the satisfaction of service providers, and service users [4], [5]. There are three information service financing schemes, namely flat fee, usage-based, and two-part tarif [6], [7].

In the issue of information service financing schemes, the utility function is one of the functions that can be applied to the problem of financing schemes [8], [9]. The quasi-linear utility function and the Stone-Geary utility function were employed in this study to gauge the degree of consumer satisfaction [5], [10]–[12]. In increasing the level of consumer satisfaction with the use of information services, apart from the utility function, it is necessary to increase marginal costs [13]–[17] and monitoring costs [18]–[21]. A marketing strategy is needed that will make the price cheaper than the total price of the packets so that it attracts consumer interest and is provided with payment options [22].

Based on this background, it is important to discuss the design of a customer preference-based internet service financing scheme [23]–[26]. The customer preference-based internet service financing scheme model is completed differentially [27]. Much research on pricing the information service, only stresses out optimization model and ignores the analytical approach [28]. So, in this discussion, the models are proved by the series of lemmas. By utilizing the functions to measure satisfaction, and make use of marginal costs, and monitoring costs, there are different schemes, and the models are observed. By customer preference, the modification model is applied to high-end and low-end as well as to heterogeneous consumers of high demand and low demand [29].

2. RESEARCH METHOD

The measures taken to complete this study are as follows:

- Describe digilib traffic data on the local server. Data was obtained from secondary data starting from January 1, 2022 to January 31, 2022 which is grouped into peak hours from 07.00 AM to 5.00 PM Indonesian time and non-peak hours from 5.01 PM to 06.59 AM.
- Define the parameters such as utility function in busy and non-busy hours, peak-time prices provided by ISPs, fees are required when following the services provided, the greatest degree of consumer in utilizing the service at peak and off-peak times, marginal and monitoring costs, consumer interest and payment options.
- Define the variables such as, service consumption levels during peak and off-peak hours, willingnes to subscribe of the customer, consumer's peak and off peak hour service consumption rates, and consumer's decision-making factor regarding participation.
- For differential solution, determine the internet service financing scheme model based on the quasi-linear utility function and the Stone-Geary utility function with the addition of marginal costs and monitoring costs as well as consumer interest, payment options, optimization consumer problems and supplier's optimization problems in flat fee, usage-based and two-part tariff financing schemes for heterogeneous consumers.
- Apply the modified model for the internet service financing scheme obtained from steps 5 by applying it to local server data already processed in step 1 and step 2.
- Complete the model in step 5 until the optimal solution is obtained. The optimization solution was assisted with LINGO 13.0 software.
- Compare the results from step 6 to obtain the optimal financing scheme for each type of consumer.

3. RESULTS AND DISCUSSION

The quasi-linear utility function and the stone-geary utility function, along with additional expenses for monitoring, consumer interest, and payment alternatives, are used in this chapter's discussion of the modification model.

3.1. Formulation of parameters dan variables

The parameters and design variables are presented in Tables 1 and 2, as follows. Tables 3 and 4 basically describe the parameters set up for the model. In Table 3 and 4, g_1/γ_1 and g_2/γ_2 are service constants during peak hours whereas h_1/β_1 and h_2/β_2 are service constants during non-peak hours where the value of g and h are determined on condition *of* g_1, g_2, h_1, and h_2 positive integers, and $g_1 > h_1, g_2 > h_2, g_1 > g_2$ and $h_1 > h_2$ for a diverse group of high-end and low-end customers. The values of g is for heterogeneous consumers with high-demand and low-demand h is determined under the condition g_1, g_2, h_1 and h_2 positive integers and $g > h$, $g_1 = g_2 = g$ and $h_1 = h_2 = h$.

Table 1. Parameters of each financing model

Parameters of each modification model	
$U(E_a, W_a)$	Utility function of E_a for busy hours and W_a for non-busy hours.
D_E	Peak-time prices provided by ISPs.
D_W	Prices offered by ISPs outside of peak times.
D	Fees are required when following the services provided.
$\bar{E}_a$	The greatest degree of consumer *a* in utilizing the service at peak times.
$\bar{W}_a$	The greatest degree of consumer *a* in utilizing the service outside of peak times.
c	Marginal costs.
t	Monitoring costs.
x	Consumer interest.
y	Payment options.

Table 2. Variables of each financing model

Variables of each modification model	
E_a	Service consumption levels during peak hours.
W_a	Service consumption levels during off-peak hours.
I_a, I_a^*	If a customer decides not to subscribe, a variable with a value of 1 indicates that they do not want to do so and consumer a's decision-making factor regarding participation, respectively.
E_a^*	Consumer a's peak hour service consumption rate.
W_a^*	Consumer a's use of services at off-peak hours.

Table 3. Parameter values of redesigned models

Parameters	Value		
	Flat fee	Usage-based	Two-part tariff
$\bar{E}_1$	25.94742	25.94742	25.94742
$\bar{E}_2$	11.22755	11.22755	11.22755
$\bar{W}_1$	10.95491	10.95491	10.95491
$\bar{W}_2$	9.952225	9.952225	9.952225
c	$0 < c < 10$	$0 < c < 10$	$0 < c < 10$
t	-	$0 < t < 10$	$0 < t < 10$
p = q	1	1	1
x = y	0.01	0.01	0.01

Table 4. Parameter values for heterogeneous consumers

Parameters	Diverse high-end and low-end consumers	Consumer heterogeneous of high-demand and low-demand
g_1/γ_1	4	3
g_2/γ_2	3	3
h_1/β_1	3	2
h_2/β_2	2	2

3.2. Heterogeneous consumers modification model for quasi-linear utility function

For the financing plan with flat fees. Optimization of the customer's issue will be as follows:

$$\underset{E_a, W_a, I_a}{\text{Max}} g_a E_a + f(W_a) - D_E E_a - D_W W_a - DI_a - (E_a + W_a)c - (E_a + W_a)x - (E_a + W_a)y$$

Subject to $E_a \leq \bar{E} I_a$, $W_a \leq \bar{W} I_a$

$$g_a E_a + f(W_a) - D_E E_a - D_W W_a - DI_a - (E_a + W_a)c - (E_a + W_a)x - (E_a + W_a)y \geq 0, I_a = 0 \text{ or } 1.$$

Optimization of the supplier problem will be as follows:

$$\underset{D, D_E, D_W}{\text{Max}} p(D_E E_1^* + D_W W_1^* + DI_1^*) + q(D_E D_2^* + D_W W_2^* + DI_2^*)$$

with $(E_a^*, W_a^*, I_a^*) = \text{argmax}\, g_a E_a + f(W_a) - D_E E_a - D_W W_a - DI_a$
subject to $E_a \leq \bar{E} I_a$, $W_a \leq \bar{W} I_a$

$$g_a E_a + f(W_a) - D_E E_a - D_W W_a - DI_a \geq 0, I_i = 0 \text{ or } 1$$

for financing plans based on consumption and two-part tariffs. Then, the optimization of the customer's issue is as follows:

$$\underset{E_a, W_a, I_a}{\text{Max}} g_a E_a + f(W_a) - D_E E_a - D_W W_a - DI_a - (c+t)E_a - (c+t)W_a - (x+y)E_a - (x+y)W_a$$

subject to $E_a \leq \bar{E} I_a$, $W_a \leq \bar{W} I_a$.

$$g_a E_a + f(W_a) - D_E E_a - D_W W_a - DI_a - (c+t)E_a - (c+t)W_a - (x+y)E_a - (x+y)W_a \geq 0$$
$I_a = 0$ or 1

Optimization of supplier problem is as follows:

$$\underset{D,D_E,D_W}{\text{Max}}\ p(D_E E_1^* + D_W W_1^* + DI_1^*) + q(D_E E_2^* + D_W W_2^* + DI_2^*)$$

With $(E_a^*, W_a^*, I_a^*) = \text{argmax}\ g_a E_a + f(W_a) - D_E E_a - D_W W_a - DI_a$
Subject to $E_a \leq \bar{E} I_a$, $W_a \leq \bar{W} I_a$

$$g_a E_a + f(W_a) - D_E E_a - D_W W_a - DI_a \geq 0,\ I_i = 0 \text{ or } 1$$

3.2.1. High-end and low-end heterogeneous consumers' modified models

Case 1a: If the ISP employs a financing method with a flat cost, it $D_E = 0, D_W = 0$, and $D > 0$. When the ISP's price has no bearing on whether or not a customer uses a service during peak or off-peak hours, the user selects the maximum consumption level of $E_1 = \bar{E}, W_1 = \bar{W}, E_2 = \bar{E}$, and $W_2 = \bar{W}$. Maximize the function on consumer problem optimization so that it is obtained $D = g_a \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y$. Thus, each high-end heterogeneous consumer is charged no more than $g_1 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y$, and low-end heterogeneous consumers no more than $g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y$.

Case 1a uses a flat-cost financing plan to balance D for both categories of consumers. If stipulated $g > g_2$ then the price for the costs of low-end heterogeneous consumers follows, resulting in the provision of high-end heterogeneous consumer costs $g_1(p) < g_2(p+q) \Leftrightarrow g_1 < \frac{g_2(p+q)}{p}$ meaning if the consumer is charged $g_1 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y$ then the cost for serving low-end heterogeneous consumers is followed by the price for serving high-end heterogeneous consumers so that. The price for the costs of low-end heterogeneous customers is then followed by the price for the provision of high-end heterogeneous consumers costs so that $g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y$, subsequently, both categories of customers can use the service. In this case for the optimization of the supplier's problems namely.

$$\underset{G}{\text{Max}} x(GH_1^*) + y(GH_2^*) = p(g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y) + q(g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y) = (p+q)(g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y)$$

Lemma 1a: If the ISP employs a flat fee financing method, the cost is:

$$D = g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y$$

with the maximum profit obtained is:

$$(p+q)(g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y)$$

Case 2a: if the ISP uses a usage-based financing scheme, it is set $D_E > 0, D_W > 0$, and $D = 0$ then, the problem of optimization for high end heterogeneous consumers. Functions on the optimization of consumer problems into $\underset{\text{E,W,I}}{\text{Max}} g_1 E_1 + f(W_1) - D_E E_1 - D_W W_1 - (c+t)E_1 - (c+t)W_1 - (x+y)E_1 - (x+y)W_1$. To maximize the function of optimizing heterogeneous consumer problems, the high-end is carried out by the differentiation of E_1 and W_1, provided that $\frac{\partial F}{\partial E_1} = 0$ and $\frac{\partial F}{\partial W_1} = 0$,

$$\Leftrightarrow \frac{\partial(g_1 E_1 + f(W_1) - D_E E_1 - D_W W_1 - (c+t)E_1 - (c+t)W_1 - (x+y)E_1 - (x+y)W_1)}{\partial E_1} = 0 \Leftrightarrow g_1 - (c+t) - (x+y) = D_E \Leftrightarrow E_1^* = \bar{E}$$

and

$$\Leftrightarrow \frac{\partial(g_1 E_1 + f(W_1) - D_E E_1 - D_W W_1 - (c+t)E_1 - (c+t)W_1 - (x+y)E_1 - (x+y)W_1)}{\partial W_1} = 0 \Leftrightarrow f'(W_1) - (c+t) - (x+y) = D_W \Leftrightarrow W_1^* = \bar{W}$$

Problem of optimization for low-end heterogeneous consumers. The objective function on the optimization of consumer problem is then:

$$\underset{E,W,I}{\text{Max}} g_2 E_2 + f(W_2) - D_E E_2 - D_W W_2 - (c+t)E_2 - (c+t)W_2 - (x+y)E_2 - (x+y)W_2$$

To maximize the function of optimizing heterogeneous consumer problem, the low-end is carried out by the differentiation of E_2 and W_2, provided that $\frac{\partial F}{\partial E_2} = 0$ and $\frac{\partial F}{\partial W_2} = 0$,

$$\Leftrightarrow \frac{\partial(g_2E_2+f(W_2)-D_EE_2-D_WW_2-(c+t)E_2-(c+t)W_2-(x+y)E_2-(x+y)W_2)}{\partial E_2} = 0 \Leftrightarrow g_2-(c+t)-(x+y) = D_E \Leftrightarrow E_2^* = \bar{E}$$

and

$$\Leftrightarrow \frac{\partial(g_2E_2+f(W_2)-D_EE_2-D_WW_2-(c+t)E_2-(c+t)W_2-(x+y)E_2-(x+y)W_2)}{\partial W_2} = 0 \Leftrightarrow f'(W_2)-(c+t)-(x+y) = D_W \Leftrightarrow W_2^* = \bar{W}$$

Optimization of supplier problems will be:

$$\underset{D_E,D_W}{Max}\, p(D_EE_1^*+D_WW_1^*)+q(D_EE_2^*+D_WW_2^*) = p(D_E\bar{E}+D_W\bar{W})+q(D_E\bar{E}+D_W\bar{W}) = p(g_1\bar{E}+\bar{W}f'(\bar{W})-(c+t)\bar{E}-(c+t)\bar{W}-(x+y)\bar{E}-(x+y)\bar{W})+q(g_2\bar{E}+\bar{W}f'(\bar{W})-(c+t)\bar{E}-(c+t)\bar{W}-(x+y)\bar{E}-(x+y)\bar{W})$$

The ISP must maximize the objective function if applied to issues during peak hours D_E and hence the best price D_E cannot be greater than $g_1-(c+t)-(x+y)$. If the ISP sets the price below $g_2-(c+t)-(x+y)$, the profit is not at its best. Applied to problems at off-peak hours, the most affordable pricing $D_W \le f'(W_1)-(c+t)-(x+y)$. If the ISP sets the price below $f'(W_2)-(c+t)-(x+y)$, then the profit is not optimal when $W_1^* \le \bar{W}$ and $W_2^* \le \bar{W}$. So, the best D_W price is $f'(W_2)-(c+t)-(x+y) \le D_W \le f'(W_1)-(c+t)-(x+y)$. Thus, the optimal price given for peak hours is $D_E = a_2-(c+t)-(x+y)$ and the optimal price given for non-peak hours is $D_W = f'(\bar{W})-(c+t)-(x+y)$, the maximum profit is $(p+q)(g_2\bar{E}+\bar{W}f'(\bar{W})-(c+t)\bar{E}-(c+t)\bar{W}-(x+y)\bar{E}-(x+y)\bar{W})$.

Lemma 2a: the ideal price charged during peak hours if the ISP implements a usage-based financing system is $D_E = a_2-(c+t)-(x+y)$ and in non-peak hours is $D_W = f'(\bar{W})-(c+t)-(x+y)$ with the maximum profit obtained being $(p+q)(g_2\bar{E}+\bar{W}f'(\bar{W})-(c+t)\bar{E}-(c+t)\bar{W}-(x+y)\bar{E}-(x+y)\bar{W})$.

Case 3a: if the ISP uses a two-part tariff financing scheme, it $D_E > 0, D_W > 0$, and $D > 0$. If it is set by $g_1 > g_2$ then it can be assumed that $g_1(p) < g_2(p+q) \Leftrightarrow g_1 < \frac{g_2(p+q)}{p}$ means that if the consumer is charged $D_E = a_1-(c+t)-(x+y)$, $D_W = f'(W_1)-(c+t)-(x+y)$ and $D = f(\bar{W})-\bar{W}f'(\bar{W})$ then only affluent, diverse consumers can use this service. If a customer gets billed by $D_E = g_2-(c+t)-(x+y)$, $D_W = f'(W_2)-(c+t)-(x+y)$, and $D = f(\bar{W})-\bar{W}f'(\bar{W})$ then heterogeneous high-end and low-end consumers can follow the service. Optimization of supplier problems into $\underset{D,D_E,D_W}{\text{Max}}\, p(D_EE_1^*+D_WW_1^*+DI_1^*)+q(D_EE_2^*+D_WW_2^*+DI_2^*) = (p+q)(g_2\bar{E}+f(\bar{W})-(c+t)\bar{E}-(c+t)\bar{W}-(x+y)\bar{E}-(x+y)\bar{W})$.

Lemma 3a: if the ISP employs a two-part tariff financing plan, the ideal D_E and D_W will be $D_E = g_2-(c+t)-(x+y)$, $D_W = f'(\bar{W})-(c+t)-(x+y)$ and $D = f(\bar{W})-\bar{W}f'(\bar{W})$ with the maximum profit obtained is $(p+q)(g_2\bar{E}+f(\bar{W})-(c+t)\bar{E}-(c+t)\bar{W}-(x+y)\bar{E}-(x+y)\bar{W})$.

3.3.2. Modification models for high-demand and low-demand heterogeneous consumers

The next lemma was discovered using similar evidence for the following three lemmas.

Lemma 4a: if the ISP employs a flat rate financing method, the cost $g\bar{E}_2+f(\bar{W}_2)-(\bar{E}_2+\bar{W}_2)c-(\bar{E}_2+\bar{W}_2)x-(\bar{E}_2+\bar{W}_2)y$ with the maximum profit obtained is $(p+q)(g\bar{E}_2+f(\bar{W}_2)-(\bar{E}_2+\bar{W}_2)c-(\bar{E}_2+\bar{W}_2)x-(\bar{E}_2+\bar{W}_2)y)$.

Lemma 5a: the ideal price during peak hours if the ISP implements a usage-based financing system is $D_E = g-(c+t)-(x+y)$ and the optimal price in non-peak hours is $D_W = f'(W_2)-(c+t)-(x+y)$ with the maximum profit obtained $(p+q)(g\bar{E}_2+\bar{W}_2f'(W_2)-(c+t)\bar{E}_2-(c+t)\bar{W}_2-(x+y)\bar{E}_2-(x+y)\bar{W}_2)$.

Lemma 6a: the best pricing is attained if the ISP uses a two-part tariff financing arrangement $D_E = g-(c+t)-(x+y)$, $D_W = f'(\bar{W}_2)-(c+t)-(x+y)$ and $D = f(\bar{W}_2)-\bar{W}_2f'(\bar{W}_2)$ with the maximum profit obtained is $p(g\bar{E}_1+f(\bar{W}_2)+\bar{W}_1f'(\bar{W}_2)-\bar{W}_2f'(\bar{W}_2)-(c+t)\bar{E}_1-(c+t)\bar{W}_1-(x+y)\bar{E}_1-(x+y)\bar{W}_1)+q(g\bar{E}_2+f(\bar{W}_2)-(c+t)\bar{E}_2-(c+t)\bar{W}_2-(x+y)\bar{E}_2-(x+y)\bar{W}_2)$.

3.4. Modified heterogeneous consumers model used for stone-geary utility function

For the flat fee financing scheme, the optimization of consumer problems will be:

$$\max_{E_a, W_a, I_a} (E_a - \gamma_a)^{\beta_a} + (W_a - \gamma_a)^{\beta_a} - D_E E_a - D_W W_a - D I_a - (E_a + W_a)c - (E_a + W_a)x - (E_a + W_a)y$$

subject to $E_a \leq \bar{E} I_a, W_a \leq \bar{W} I_a$.

$$(E_a - \gamma_a)^{\beta_a} + (W_a - \gamma_a)^{\beta_a} - D_E E_a - D_W W_a - D I_a - (E_a + W_a)c - (E_a + W_a)x - (E_a + W_a)y \geq 0$$
$I_a = 0$ or 1

Optimization of supplier problems is as follows:

$$\max_{D, D_E, D_W} p(D_E E_1^* + D_W W_1^* + D I_1^*) + q(D_E D_2^* + D_W W_2^* + D I_2^*)$$

with $(E_a^*, W_a^*, I_a^*) = \text{argmax}\,(E_a - \gamma_a)^{\beta_a} + (W_a - \gamma_a)^{\beta_a} - D_E E_a - D_W W_a - D I_a$
subject to $E_a \leq \bar{E} I_a$, $W_a \leq \bar{W} I_a$, $(E_a - \gamma_a)^{\beta_a} + (W_a - \gamma_a)^{\beta_a} - D_E E_a - D_W W_a - D I_a \geq 0$, $I_i = 0$ or 1.
To finance usage-based and two-part tariff schemes then optimization of customer issues is as follows:

$$\max_{E_a, W_a, I_a} (E_a - \gamma_a)^{\beta_a} + (W_a - \gamma_a)^{\beta_a} - D_E E_a - D_W W_a - D I_a - (c + t)E_a - (c + t)W_a - (x + y)E_a - (x + y)W_a$$

subject to $E_a \leq \bar{E} I_a, W_a \leq \bar{W} I_a$

$$(E_a - \gamma_a)^{\beta_a} + (W_a - \gamma_a)^{\beta_a} - D_E E_a - D_W W_a - D I_a - (c + t)E_a - (c + t)W_a - (x + y)E_a - (x + y)W_a \geq 0, I_a = 0 \text{ or } 1.$$

Optimization of supplier problems will be:

$$\max_{D, D_E, D_W} p(D_E E_1^* + D_W W_1^* + D I_1^*) + q(D_E E_2^* + D_W W_2^* + D I_2^*)$$

with $(E_a^*, W_a^*, I_a^*) = \text{argmax}\,(E_a - \gamma_a)^{\beta_a} + (W_a - \gamma_a)^{\beta_a} - D_E E_a - D_W W_a - D I_a$
subject to $E_a \leq \bar{E} I_a$, $W_a \leq \bar{W} I_a$, $(E_a - \gamma_a)^{\beta_a} + (W_a - \gamma_a)^{\beta_a} - D_E E_a - D_W W_a - D I_a \geq 0$, $I_i = 0$ or 1.

3.4.1. High-end and low-end heterogeneous consumers' modified models

Using related proofs for the subsequent three lemmas.
Lemma 1b: if an internet service provider (ISP) adopts a flat rate financing method, the cost to users becomes $G = (\bar{E} - \gamma_2)^{\beta_2} + (\bar{W} - \gamma_2)^{\beta_2} - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y$ and the maximum profit earned is $(p + q)\big((\bar{P} - \gamma_2)^{\beta_2}(\bar{E} - \gamma_2)^{\beta_2} + (\bar{W} - \gamma_2)^{\beta_2} - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y\big)$.
Lemma 2b: the ideal price is if the ISP employs a usage-based financing system $D_E = \beta_2(\bar{E} - \gamma_2)^{\beta_2 - 1} - (c + t) - (x + y)$ and $D_W = \beta_2(\bar{W} - \gamma_2)^{\beta_2 - 1} - (c + t) - (x + y)$ with the maximum profit obtained is $(p + q)\Big(\big(\beta_2(\bar{E} - \gamma_2)^{\beta_2 - 1}\bar{E} - (c + t)\bar{E} - (x + y)\bar{E}\big) + \big(\beta_2(\bar{W} - \gamma_2)^{\beta_2 - 1}\bar{W} - (c + t)\bar{W} - (x + y)\bar{W}\big)\Big)$.
Lemma 3b: the ideal pricing will be if the ISP employs a two-part tariff financing system, which is $D_E = \beta_2(\bar{E} - \gamma_2)^{\beta_2 - 1} - (c + t) - (x + y)$ and $D_W = \beta_2(\bar{W} - \gamma_2)^{\beta_2 - 1} - (c + t) - (x + y)$ and $D = 0$ where the maximum profit obtained is $(p + q)\big(\beta_2(\bar{E} - \gamma_2)^{\beta_2 - 1}\bar{E} + \beta_2(\bar{W} - \gamma_2)^{\beta_2 - 1}\bar{W} - (c + t)\bar{E} - (c + t)\bar{W} - (x + y)\bar{E} - (x + y)\bar{W}\big)$.

3.4.2. Modification models in heterogeneous consumers of high demand and low demand

Using the same supporting evidence for the subsequent three lemmas then we have as follows.
Lemma 4b: the fee paid becomes a flat fee if the ISP adopts this type of financing $D = (\bar{E}_2 - \gamma)^{\beta} + (\bar{W}_2 - \gamma)^{\beta} - (\bar{E}_2 + \bar{W}_2)c - (\bar{E}_2 + \bar{W}_2)x - (\bar{E}_2 + \bar{W}_2)y$ and the maximum profit obtained is $(p + q)\big((\bar{E}_2 - \gamma)^{\beta} + (\bar{W}_2 - \gamma)^{\beta} - (\bar{E}_2 + \bar{W}_2)c - (\bar{E}_2 + \bar{W}_2)x - (\bar{E}_2 + \bar{W}_2)y\big)$.
Lemma 5b: the ideal pricing is, if the ISP employs a usage-based finance scheme, $D_E = \beta(\bar{E}_2 - \gamma)^{\beta - 1} - (c + t) - (x + y)$, and $D_W = \beta(\bar{W}_2 - \gamma)^{\beta - 1} - (c + t) - (x + y)$ with the maximum profit obtained is $p\big(\beta(\bar{E}_1 - \gamma)^{\beta - 1}\bar{E}_1 + \beta(\bar{W}_1 - \gamma)^{\beta - 1}\bar{W}_1 - (c + t)\bar{E}_1 - (c + t)\bar{W}_1 - (x + y)\bar{E}_1 - (x + y)\bar{W}_1\big) +$
$q\big(\beta(\bar{E}_2 - \gamma)^{\beta - 1}\bar{E}_2 + \beta(\bar{W}_2 - \gamma)^{\beta - 1}\bar{W}_2 - (c + t)\bar{E}_2 - (c + t)\bar{W}_2 - (x + y)\bar{E}_2 - (x + y)\bar{W}_2\big)$.
Lemma 6b: the ideal pricing will be if the ISP employs a two-part tariff financing arrangement $D_E = \beta(\bar{E}_2 - \gamma)^{\beta - 1} - (c + t) - (x + y)$, and $D_W = \beta(\bar{W}_2 - \gamma)^{\beta - 1} - (c + t) - (x + y)$ with the maximum profit

obtained is $p\big((\bar{E}_2-\gamma)^\beta+(\bar{W}_2-\gamma)^\beta+\beta(\bar{E}_2-\gamma)^{\beta-1}\bar{E}_1+\beta(\bar{W}_2-\gamma)^{\beta-1}\bar{W}_1-\beta(\bar{E}_2-\gamma)^{\beta-1}\bar{E}_2-\beta(\bar{W}_2-\gamma)^{\beta-1}\bar{W}_2-(c+t)\bar{E}_1-(c+t)\bar{W}_1-(x+y)\bar{E}_1-(x+y)\bar{W}_1\big)+q\big((\bar{E}_2-\gamma)^\beta+(\bar{W}_2-\gamma)^\beta-(c+t)\bar{E}_2-(c+t)\bar{W}_2-(x+y)\bar{E}_2-(x+y)\bar{W}_2\big)$.

3.5. Optimal financing scheme for heterogeneous consumers

The following are the results obtain based on calculations performed for a diverse group of consumers. Based on Table 5, the maximum profit of the two utility functions is in the stone-geary utility function with a profit of IDR 2727.77269 per kbps and profit of IDR 1685.95079 per kbps for high-demand and low-demand customers.

Table 5. Maximum advantages for high-end and low-end and high-demand and low-demand heterogeneous consumers

Scheme	Quasi-linear	Stone-geary	Quasi-linear	Stone-geary
	High-end and low-end		High-demand low-demand	
Flat fee	393.490	1177.515	264.18807	115.39062
Usage-based	632.772	2727.772	461.85804	1685.95079
Two-part tariff	392.752		327.25309	485.88746

4. CONCLUSION

Based on the findings and subsequent discussion, it can be concluded that a usage-based financing structure combined with a two-part tariff of IDR 2727.77269 per kbps yields the highest profit from the customer preference-based internet service financing scheme model for heterogeneous consumers (high end and low end as well as high and low demand). Basically, flat fee and two-part tariff schemes yield slightly different objective function values for all schemes. For further research, it is suggested to focus on the improvement of models (lemma by also taking care of consumers' ability) to automatically transform into another scheme as they prefer to.

ACKNOWLEDGEMENT

The research/publication of this article was funded by DIPA of Public Service Agency of Universitas Sriwijaya 2022. SP DIPA-023.17.2.677515/2022, On December 13, 2021. In accordance with the Rector's Decree Number: 0109/UN9.3.3.1/SK/2023, on April 2022.

REFERENCES

[1] R. Amin, "Analytical hierarchy process method in internet service provider selection decision support system (in Indonesian: metode analytical hierarchy process dalam sistem pendukung keputusan pemilihan internet service provider)," *J. Tek. Komput. AMIK BSI*, vol. 1, no. 1, pp. 66–71, 2015.

[2] I. Lee, "Pricing models for the internet of things (IoT): game perspectives," *Internet of Things*, vol. 15, p. 100405, Sep. 2021, doi: 10.1016/j.iot.2021.100405.

[3] G. L. Rosston and S. J. Wallsten, "Increasing low-income broadband adoption through private incentives," *Telecomm. Policy*, vol. 44, no. 9, p. 102020, 2020.

[4] Y. Li, J. Li, and M. Ahmed, "A three-stage incentive formation for optimally pricing social data offloading," *Journal of Network and Computer Applications*, vol. 172, no. 2018, p. 102816, Dec. 2020, doi: 10.1016/j.jnca.2020.102816.

[5] Z. Yang, X. Cao, F. Wang, and C. Lu, "Fortune or Prestige? The effects of content price on sales and customer satisfaction," *Journal of Business Research*, vol. 146, no. October 2020, pp. 426–435, Jul. 2022, doi: 10.1016/j.jbusres.2022.03.075.

[6] I. Indrawati, F. M. Puspita, R. Resmadona, E. Yuliza, O. Dwipurwani, and S. Octarina, "Analysis of information service pricing scheme model based on customer self-selection," *Science and Technology Indonesia*, vol. 6, no. 4, pp. 337–343, Oct. 2021, doi: 10.26554/sti.2021.6.4.337-343.

[7] I. Indrawati, F. M. Puspita, E. Yuliza, O. Dwipurwani, Y. E. Putri, and Affriyanti, "Improved cloud computing model of internet pricing schemes based on Cobb-Douglas utility function," in *Journal of Physics: Conference Series*, vol. 1282, no. 1, 2019.

[8] M. F. Quaas, S. Baumgärtner, M. A. Drupp, and J. N. Meya, "Intertemporal utility with heterogeneous goods and constant elasticity of substitution," *Economics Letters*, vol. 191, p. 109092, Jun. 2020, doi: 10.1016/j.econlet.2020.109092.

[9] Y. Liu, Y. Li, Q. Ma, S. Ioannidis, and E. Yeh, "Fair caching networks," *Performance Evaluation*, vol. 143, p. 102138, Nov. 2020, doi: 10.1016/j.peva.2020.102138.

[10] S. G. Taheri, M. Navabakhsh, H. Tohidi, and D. Mohammaditabar, "A system dynamics model for optimum time, profitability, and customer satisfaction in omni-channel retailing," *J. Retail. Consum. Serv.*, vol. 78, no. January, p. 103784, 2024.

[11] J. Li, W. Dong, and J. Ren, "The effects of user- and marketer-generated content on customer satisfaction: a textual analysis approach," *Electron. Commer. Res. Appl.*, vol. 65, no. February 2022, p. 101407, 2024.

[12] E. W. Mainardes, A. R. S. Coutinho, and H. M. B. Alves, "The influence of the ethics of E-retailers on online customer experience and customer satisfaction," *J. Retail. Consum. Serv.*, vol. 70, no. June 2022, p. 103171, 2023.

[13] M. Li, H. Feng, F. Chen, and J. Kou, "Numerical investigation on mixed bundling and pricing of information products," *International Journal of Production Economics*, vol. 144, no. 2, pp. 560–571, Aug. 2013, doi: 10.1016/j.ijpe.2013.04.015.

[14] Y. Wang, Z. Yang, J. Yu, and J. Liu, "An optimization-based partial marginal pricing method to reduce excessive consumer payment in electricity markets," *Applied Energy*, vol. 352, no. September, p. 121935, Dec. 2023, doi: 10.1016/j.apenergy.2023.121935.
[15] C. P. Barala, P. Mathuria, and R. Bhakar, "Distribution locational marginal price based hierarchical scheduling framework for grid flexibility from virtual energy storage systems," *Electric Power Systems Research*, vol. 214, no. PA, p. 108866, Jan. 2023, doi: 10.1016/j.epsr.2022.108866.
[16] Y. Tang, S. C. Perera, Q. Cao, and X. Ji, "Supplier versus platform bundling: optimal strategies under agency selling," *Transportation Research Part E: Logistics and Transportation Review*, vol. 179, no. September, p. 103325, Nov. 2023, doi: 10.1016/j.tre.2023.103325.
[17] D. Fraiman, "A self-organized criticality participative pricing mechanism for selling zero-marginal cost products," *Chaos, Solitons and Fractals*, vol. 158, p. 112028, 2022.
[18] R. Sitepu, F. M. Puspita, A. N. Pratiwi, and I. P. Novyasti, "Utility function-based pricing strategies in maximizing the information service provider's revenue with marginal and monitoring costs (double)," *International Journal of Electrical and Computer Engineering*, vol. 7, no. 2, pp. 877–887, 2017, doi: 10.11591/ijece.v7i2.pp877-887.
[19] M. Huang, X. Li, M. Yang, and X. Kuai, "Intelligent coverage and cost-effective monitoring: Bus-based mobile sensing for city air quality," *Comput. Environ. Urban Syst.*, vol. 108, no. January, p. 102073, 2024.
[20] P. Agade and E. Bean, "GatorByte – an internet of things-based low-cost, compact, and real-time water resource monitoring buoy," *HardwareX*, vol. 14, p. e00427, 2023.
[21] S. Kotsilitis, E. C. Marcoulaki, and E. Kalligeros, "A versatile, low-cost monitoring device suitable for non-intrusive load monitoring research purposes," *Meas. Sensors*, vol. 32, no. August 2023, p. 101081, 2024.
[22] B. R. Beattie and J. T. LaFrance, "The law of demand versus diminishing marginal utility," *Review of Agricultural Economics*, vol. 28, no. 2, pp. 262–271, Jun. 2006, doi: 10.1111/j.1467-9353.2006.00286.x.
[23] A. M. Ghouri, P. Akhtar, M. A. Haq, V. Mani, G. Arsenyan, and M. Meyer, "Real-time information sharing, customer orientation, and the exploration of intra-service industry differences: Malaysia as an emerging market," *Technological Forecasting and Social Change*, vol. 167, no. January 2020, p. 120684, Jun. 2021, doi: 10.1016/j.techfore.2021.120684.
[24] C. Kang and D. B. Wooten, "The Presenter's Paradox in customer service interactions," *Journal of Business Research*, vol. 120, no. July, pp. 94–102, Nov. 2020, doi: 10.1016/j.jbusres.2020.07.041.
[25] Y. Wu, Y. Wu, J. M. Guerrero, and J. C. Vasquez, "A comprehensive overview of framework for developing sustainable energy internet: From things-based energy network to services-based management system," *Renewable and Sustainable Energy Reviews*, vol. 150, no. May, p. 111409, Oct. 2021, doi: 10.1016/j.rser.2021.111409.
[26] C. Prentice and M. Nguyen, "Engaging and retaining customers with AI and employee service," *Journal of Retailing and Consumer Services*, vol. 56, no. June, p. 102186, Sep. 2020, doi: 10.1016/j.jretconser.2020.102186.
[27] R. Sitepu and F. M. Puspita, "Cobb-douglas utility function of information service pricing scheme based on monitoring and marginal costs," *International Conference on Education , Technology and Sciences*, pp. 602–608, 2016, [Online]. Available: http://repository.unsri.ac.id/98764/.
[28] P. Rita, T. Oliveira, and A. Farisa, "The impact of e-service quality and customer satisfaction on customer behavior in online shopping," *Heliyon*, vol. 5, no. 10, p. e02690, Oct. 2019, doi: 10.1016/j.heliyon.2019.e02690.
[29] F. M. Puspita, B. J. Rezky, A. N. . Sinarmata, E. Yuliza, and Y. Hartono, "Improve incentive pricing-based quasi-linier utility function of wireles networks," *Indonesian Journal of Electrical Enginering and Computer Science*, vol. 22, no. 3, pp. 1467–1475, 2021.

BIOGRAPHIES OF AUTHORS

Indrawati obtained her S.Si. degree in Mathematics from Sriwijaya University, South Sumatera, Indonesia in 1996. Then she received her M.Si. in Actuarial Science in 2004 at Bandung Institute of Technology. She has been a Mathematics Department member at The Faculty of Mathematics and Natural Sciences at Sriwijaya University South Sumatera Indonesia since 1998. She is currently a Ph.D. student in Mathematics and Natural Sciences Study Program of Sriwijaya University since 2022. Her research interests include optimization, actuarial science, insurance problems and operations research focussing on inventory problem. She can be contacted at email: indrawati@mipa.unsri.ac.id.

Fitri Maya Puspita received her S.Si. degree in Mathematics from Sriwijaya University, South Sumatera, Indonesia in 1997. Then she received her M.Sc. in Mathematics from Curtin University of Technology (CUT) Western Australia in 2004. She received her Ph.D. in Science and Technology in 2015 from Universiti Sains Islam Malaysia. She has been a Mathematics Department member at Faculty of mathematics and Natural Sciences at Sriwijaya University in South Sumatera Indonesia since 1998. Her research interests include optimization and its applications such as vehicle routing problems and QoS pricing and charging in third generation internet. She can be contacted at email: fitrimayapuspita@unsri.ac.id.

Evi Yuliza obtained her S.Si. degree in Mathematics from Sriwijaya University, South Sumatera, Indonesia in 2000. Then she received her M.Si. in Universitas Gadjah Mada in 2004. She received her Ph.D. in Mathematics and Natural Science in 2021 from Sriwijaya University. She has been a Mathematics Department member at Faculty of mathematics and Natural Sciences Sriwijaya University South Sumatera Indonesia since 2008. Her research interests include optimization, focussing on vehicle routing problems and its varians. She can be contacted at email: eviyuliza@mipa.unsri.ac.id.

Oki Dwipurwani obtained her S.Si. degree in Statitistics, from Padjadjaran University in Bangung, South Sumatera, Indonesia in 1997. Then she received her M.Si. in in 2003 from Bogor Institute of Agriculture. She has been a Mathematics Department member at Faculty mathematics and Natural Sciences at Sriwijaya University in South Sumatera Indonesia since 2000. She is currenty a Ph.D. student in Mathematics and Natural Sciences Study Program of Sriwijaya Unviersity since 2022. Her research interests include statistical and its application and operations research focussing on probabilistic inventory problem. She can be contacted at email: okidwip@unsri.ac.id.

Sisca Octarina achieved her S.Si. degree in Mathematics from Sriwijaya University, South Sumatera, Indonesia in 2005. Then she received her M.Sc. in Mathematics from Nanyang Technological University in 2010. She reveived her Ph.D. in Science in 2022 from Sriwijaya University. She has been a Mathematics Department member at Faculty of Mathematics and Natural Sciences Sriwijaya University South Sumatera Indonesia since 2006. Her research interests include optimization and its applications such as vehicle routing problem, cutting stock problems and set covering problems with all methods and applications. She can be contacted at email: sisca_octarina@unsri.ac.id.

Rizky Helmayanti has graduated from Sriwijaya University Faculty of Mathematics and Natural Sciences, in 2022. Her topic interest includes optimization and its application on pricing of information service focusing on marginal dan monitoring cost. She can be contacted at email: rizkyhelmayanti17@gmail.com.