Probabilistic inventory modeling for chlorine gas using minitab and Python: a comparative study of demand distributions

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ABSTRACT

The availability of chlorine gas (Cl_2) is a critical component in the drinking water disinfection process at the regional drinking water company (PDAM), as it plays a vital role in ensuring microbiological safety. Disruptions in the chlorine gas supply may lead to interruptions in water distribution and pose significant public health risks. This study investigates the application of a probabilistic (Q, r) inventory model for managing chlorine gas stock, incorporating several probability distributions that satisfy the underlying model assumptions. The resulting optimal inventory policies derived from each distribution are then compared. Chlorine gas demand forecasting is also performed using the seasonal autoregressive integrated moving average (SARIMA) model. The objective of this research is to generate an optimal inventory policy and accurate demand forecasts, with the entire implementation carried out in Python software. The results show that the best model obtainis the SARIMA $(0,1,0)(0,1,1)^{12}$ model, with a MAPE value of 5.48%, and that the chlorine gas demand data follow normal, gamma, exponential, and erlang probability distributions. The comparison results show that the optimal policy of the gamma probabilistic model provides the best results, as well as being better than Normal and exponential policies in previous studies.

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1. INTRODUCTION

The water purification process at the regional drinking water company (PDAM) requires several chemical substances, including aluminum sulfate, chlorine gas, lime, and calcium hypochlorite. Among these, chlorine gas is the most critical, and its continuous availability is essential. Chlorine gas serves a primary role in the disinfection process by eliminating bacteria, viruses, and other pathogenic microorganisms. In the absence of effective disinfection, drinking water may become a vector for serious waterborne diseases such as cholera, typhoid, and dysentery. A failure to provide adequate disinfectants poses an immediate risk to public health.

A study conducted by [1], [2] demonstrated that chlorine gas (Cl_2) is highly effective in reducing total coliform bacteria, reinforcing its role as the principal disinfectant in drinking water treatment. Regulatory standards mandate that drinking water must be free from pathogenic microorganisms and must meet specified quality criteria. Disinfection, particularly through chlorination, is a legally required step to ensure the microbiological safety of drinking water.

Calcium hypochlorite is often considered an alternative disinfectant to chlorine gas. While it serves a similar function, it exists in solid form, making it easier to store. However, it must be dissolved before use, and its efficacy can degrade if not stored properly. Although calcium hypochlorite can substitute chlorine gas in certain conditions, it is not the preferred option for large-scale treatment systems. Given its critical role in maintaining microbiological safety, the availability of chlorine gas must be prioritized. Any disruption in its supply can compromise water safety and potentially halt distribution operations, thereby endangering public health.

Therefore, the urgency of optimal chlorine gas inventory management is evident in PDAM operations. One suitable approach for addressing statistical inventory management problems is the statistical inventory control (SIC) method [3]. In this study, the probabilistic SIC approach, specifically the probabilistic (Q, r) inventory model, is considered more appropriate. This is because the demand for chlorine gas exhibits variability and uncertainty, making it difficult to predict with precision. However, key statistical characteristics such as the expected value, variance, and probability distribution of demand can be estimated from historical data, allowing for the application of a probabilistic model.

Research on probabilistic inventory models, specifically the economic order quantity (EOQ) or probabilistic (r, Q) models, has been conducted across various fields using demand distributions such as the normal and gamma distributions [4]–[9]. The probabilistic EOQ model under uniformly distributed demand has also been examined by [10]. Additionally, [11] compared transportation inventory models under gamma, exponential, and uniform demand distributions.

Research on chlorine gas inventory models has also been conducted by [12], who applied the EOQ model with a normally distributed demand assumption to manage chlorine gas inventory at PT Toya Indo Manunggal. Their findings indicated that the EOQ method yielded a more optimal inventory policy compared to the company's existing approach. In addition, a study by [13] examined chlorine gas inventory at PDAM Tirta Musi using the EOQ model under the assumption of exponentially distributed demand. However, there has been limited development of inventory models for chlorine gas using alternative probability distributions. Furthermore, no prior studies have analyzed the relationship between optimal inventory policies and the values of the akaike information criterion (AIC) and the bayesian information criterion (BIC) in evaluating the goodness-of-fit of probability distributions for chlorine gas demand. This study aims to compare the optimal solutions of probabilistic (Q, r) inventory models under various probability distributions that satisfy the distributional assumption tests. The analysis is conducted using Python software. Research on probabilistic inventory models (Q,r) with non-normal distributions is still rare due to the complexity of model development. This study emphasizes the development of a probabilistic inventory model at PDAM, with demand parameters for chlorine gas distributed according to a non-normal probability distribution. It then compares the results of the optimal policies from these inventory models.

In modeling inventory systems that involve the demand or consumption of chlorine gas chemicals, it is sometimes necessary to generate forecasts using appropriate time series methods. One such method is the seasonal autoregressive integrated moving average (SARIMA) model. SARIMA has been shown to provide more accurate and reliable forecasts compared to several other approaches, such as the holt-winters method [14]. A comparative study conducted by [15] on forecasting monthly river flows in western Cuba also demonstrated that SARIMA outperformed the holt-winters method in capturing seasonal and inter-annual variability. Accordingly, this research employs the SARIMA model. Moreover, SARIMA remains competitive with various machine learning forecasting techniques, as reported by [16], [17]. One application of SARIMA is forecasting demand in the fashion sector: predicting long-term sales of highly seasonal shoe models based on historical data using the Prophet and SARIMA algorithms. Here, SARIMA provides better forecasting results than Prophet [18]. Another application is platelet demand forecasting using the SARIMA model to optimize the allocation of blood bank resources and clinical supplies [19]. To date, PDAM has never estimated chlorine gas demand, leading to excessive orders.

2. RESEARCH METHOD

2.1. SARIMA forecasting method

The general form of the SARIMA $(p, d, q)(P, D, Q)^L$ can be expressed as (1):

$$\Phi_p(B^L)\phi_p(B)(1-B)^d(1-B^S)^D X_t = \theta_q(B)\Theta_Q(B^L)e_t \tag{1}$$

Here, p, d, and q represent the non-seasonal AR, differencing, and MA orders, respectively, and P, D, and Q represent the seasonal AR, differencing, and MA orders, respectively. $\phi_p(B)$ denotes the non-seasonal AR component, $\Phi_p(B^L)$ denotes the seasonal AR component, $(1-B)^{\mathrm{d}}$ is the non-seasonal differencing operator,

$(1-B^L)^D$ is the seasonal differencing operator, $\theta_q(B)$ denotes the non-seasonal MA component, $\Theta_Q(B^L)$ denotes the seasonal MA component, and e_t represents the error term at period t [20].

Data stationarity testing must be performed first, including tests for mean and variance stationarity. If the data does not meet the data stationarity requirements, transformation and differentiation must be performed. If the data is stationary, the next step is to plot the autocorrelation function (ACF) and partial autocorrelation function (PACF) for SARIMA model identification. Estimation of SARIMA $(p,d,q)(P,D,Q)^L$ model coefficients can be estimated using the maximum Likelihood method [21].

$$L(\phi,\mu,\sigma_\varepsilon^2) = (2\pi\sigma_\varepsilon^2)^{-n/2}(1-\phi^2)^{1/2}exp[-S(\phi,\mu)/2\sigma_\varepsilon^2] \tag{2}$$

where n is the number of observations or data. ϕ is the autoregressive parameter, μ is the mean of the stationary process, and σ_ε^2 is the variance of the error. $S(\phi,\mu)$ is a quantity representing the sum of squares. Finally, verify the SARIMA models obtained based on ACF and PACF values using mean absolute percentage error (MAPE). The SARIMA model with the smallest MAPE value is selected for the forecasting and probabilistic inventory modeling stages.

According to [22], the error in time series data analysis can be calculated using the MAPE. MAPE assesses the magnitude of the difference between forecasted values and actual observed data. The equation for calculating MAPE is given as (3):

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{|Z_t-\hat{Z}_t|}{Z_t} \times 100\% \tag{3}$$

where n represents the total number of data points, Z_t is the actual data in period t, and $\hat{Z}_t$ is the forecasted data for period ttt. The criteria for interpreting the MAPE value are as follows: excellent for MAPE < 10%, good for 10% ≤ MAPE < 20%, fair for 20% ≤ MAPE < 50%, and poor for MAPE ≥ 50%.

2.2. Parameters and variables of the probabilistic inventory model

The parameters and variables used in this research model are presented in detail in Table 1. This presentation aims to clarify the main components that make up the model, thereby facilitating an understanding of the model's structure and mechanisms. In addition, the Table 1 also helps identify the relationships between variables and the role of each parameter in the analysis process.

Table 1. Defining variables and parameters

Variables and parameters	The defining variables and parameters
η	Service level, representing the probability that demand can be fulfilled without experiencing a stockout.
N	Expected inventory shortage per cycle, i.e., the amount of demand that cannot be fulfilled.
D_L	Expected demand during the lead time period.
α	Proportion of unfulfilled demand, where η=1-α.
r	Reorder point, the inventory level at which a new order is placed
X	Random variable representing the demand for chlorine gas.
$f(x)$	Probability density function of demand.
D	Expected demand over the planning horizon (kg/year).
L	Lead time, defined as the time interval from placing an order until the goods arrive (years).
Q	Order quantity or lot size for each replenishment (kg).
p	Unit price per kilogram.
A	Ordering or communication cost per order (IDR per message).
h	Holding cost per unit (percentage per unit per year), calculated as a percentage of the unit price and proportional to the quantity stored and the storage duration.
C_u	Unit shortage cost (IDR per unit), proportional to the number of units that cannot be fulfilled.
Tc	Total cost of the inventory system.
ss	Safety stock, representing the buffer quantity kept in the warehouse to prevent stockouts.

2.3. Normal probabilistic inventory model

Suppose the demand for chemicals follows a normal probability distribution, with a corresponding normal probability density function *f*(*x*). In that case, the order quantity per cycle (*Q*), the reorder point (*r*), and the expected number of inventory shortages (*N*) are formulated as described in the (4):

$$r = D_L + Z_\alpha S_L \tag{4}$$

$$Q = \sqrt{\frac{2D(A+C_u N)}{ph}} \tag{5}$$

$$N = \int_r^\infty (x-r) f(x) dx = S_L[f(Z_\alpha) - Z_\alpha \Psi(Z_\alpha)] \tag{6}$$

where Z_α represents the standardized normal value. The notation $f(Z_\alpha)$ and $\Psi(Z_\alpha)$ denote the probability density function (PDF) and cumulative distribution function (CDF) of the standard normal distribution, respectively [23]. The total cost is calculated as (7):

$$O_T = Dp + \left(\frac{A}{T}\right) + h\left(R - D_L + \frac{DT}{2}\right) + \frac{C_u}{T}\int_R^\infty (z-R) f(z) dz \tag{7}$$

2.4. Testing the probability distribution of data statistics with kolmogorov-smirnov (KS)

To determine the statistical probability distribution of the demand data, the kolmogorov–smirnov (KS) test statistic is used [24]. If X_i represents the ordered residual data from the smallest to the largest value, the KS statistic can be calculated using (8):

$$KS\_value = sup_x |F_n(x) - F_0(x)| \tag{8}$$

where $F_n(x)$ denotes the cumulative distribution function (CDF) of the specified theoretical distribution, and $F_0(x)$ represents the empirical cumulative distribution function (ECDF) of the observed data. The hypotheses are defined as follows: H_0: the residuals follow the specified distribution, and H_1 : The residuals do not follow the specified distribution. The null hypothesis H_0 is rejected if the KS test statistic exceeds the critical value, i.e., $KS\_value > KS_{\alpha,n}$, or if the *p*-value is less than the significance level, i.e., $KS\_pvalue < \alpha$ [25], [26]

Alongside the KS statistic, one can assess the goodness-of-fit of a probability distribution through the AIC and the BIC. Their formulations are as (9) and (10):

$$AIC = -2\log(L) + 2(p+1) \tag{9}$$

$$BIC = -2\log(L) + (p+1)\log(n) \tag{10}$$

where L denotes the estimated likelihood function, p is the number of parameters in the model, and *n* is the number of observations in the dataset [27].

2.5. Hadley-within algorithm

The Haddley-Within algorithm for obtaining the optimal *Q* and *r* values in the (*Q*, *r*) model has several stages [28]. First, determine the initial order quantity per cycle (Q_0). Second, find the probability of inventory shortage (α). In this second stage, the reorder point (*r*) value will be obtained. The third step is to enter the *r* value obtained in the second step into the order quantity per cycle (*Q*) in (4). The fourth step is an iteration in which the new α and *r* values are recalculated using the same formula, then returning to Step 2. At this stage, if a new *r* value is obtained that is relatively close to the previous iteration's *r*, then the iteration is complete. The *r*-value for the *Q*-value in this step is the optimal value. However, if it is still significantly different, return to the second step, starting from the last *r* and *Q* values obtained. The Hadley-Within algorithm was chosen for its simplicity and ability to accommodate the optimization of probabilistic inventory models that will be developed with non-normally distributed demand.

2.6. Research method

Figure 1 presents a flowchart illustrating the overall research methodology, specifically focusing on the application of the SARIMA forecasting method and the *(Q,r)* probabilistic inventory model. This flowchart provides a systematic overview of the research stages, from data processing to the analysis of results. Additionally, this visualization helps clarify the relationships between processes and facilitates understanding of the implementation flow of the methods used in this study.

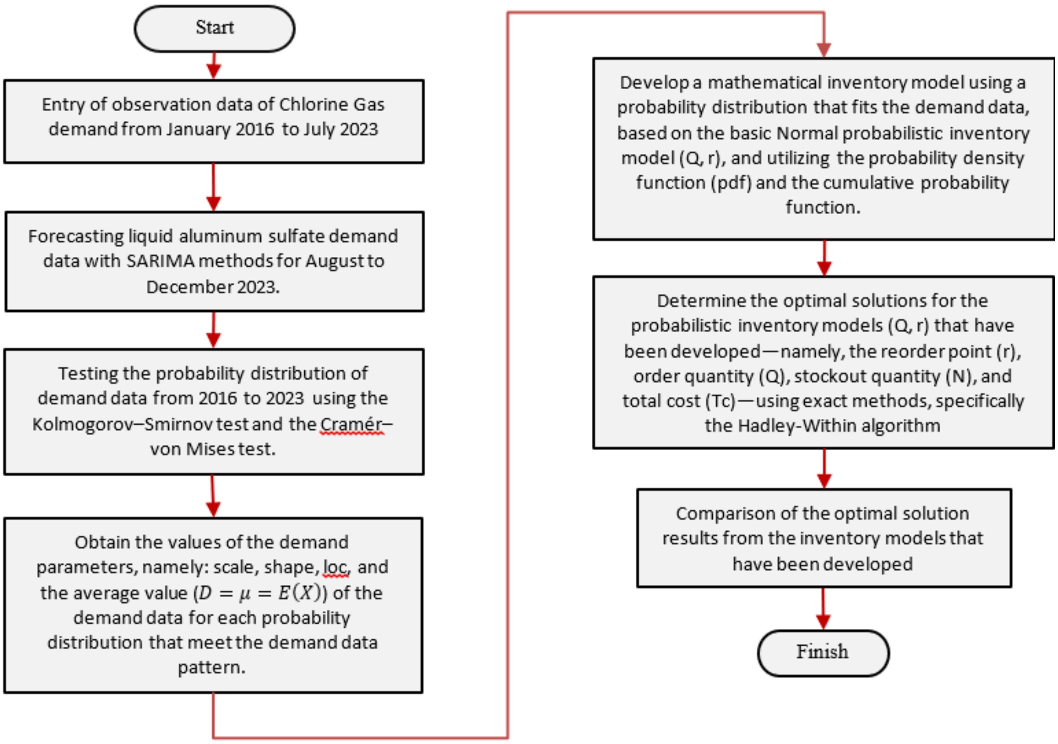

Figure 1. Flowchart of research method

3. RESULTS AND DISCUSSION

3.1. SARIMA

A graphical representation of the chlorine gas demand data is presented in Figure 2, which illustrates the volatility observed in the time series from January 2016 to July 2023. Forecasting for the period from August to December 2023 is necessary, as the inventory modeling process requires complete annual average demand data.

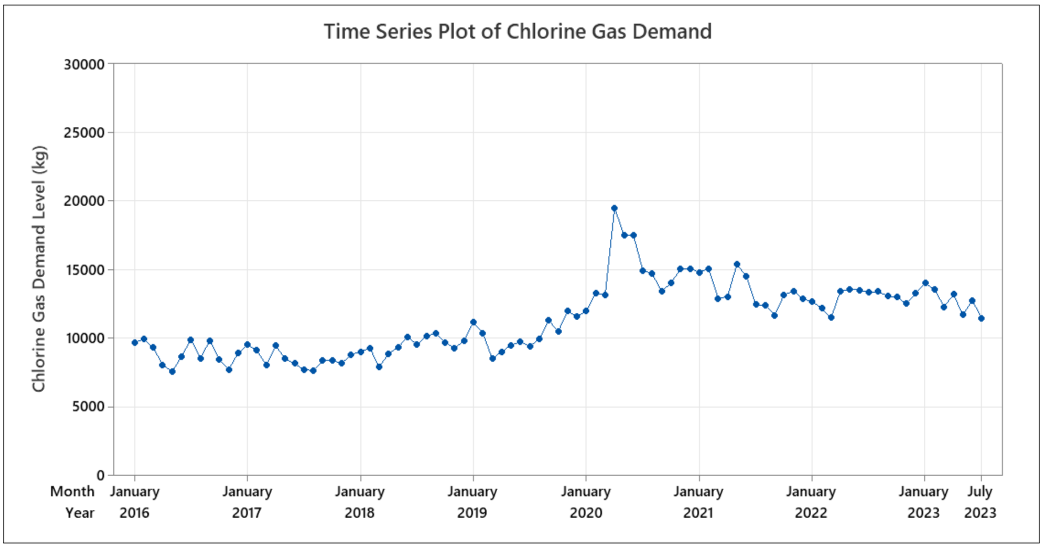

Figure 2. Time series graph of chlorine gas demand from January 2016 to July 2023

The dataset is divided into two segments: the training data, used to develop the SARIMA model, consists of time series data from January 2016 to December 2021; while the testing data, used to evaluate the model's performance, includes data from January 2022 to July 2023. The data from a single differentiation ($d = 1$) provides ACF and PACF values, none of which are significant. This means that in identifying the SARIMA model, it will be necessary to try combinations of $p = \{0,1\}$, $q = \{0,1\}$, $P = \{0,1\}$, and $Q = \{0,1\}$, resulting in a total of 16 combinations for the parameter values p, q, P, and Q that need to be tested. The optimal model identified is SARIMA $(0,1,0)\ (0,1,1)^{12}$, which yields a MAPE of 5.48%, as calculated using in (3). This MAPE value indicates that the SARIMA $(0,1,0)\ (0,1,1)^{12}$ model demonstrates very high predictive accuracy. The plots of the testing data and the forecast are presented in Figure 3. Data preprocessing and modeling using the SARIMA approach in Minitab software. In Figure 3, the blue line represents the actual chlorine gas data from January 2022 to July 2023, while the red line illustrates the predicted and forecasted values from January 2022 to December 2023. The green and purple lines indicate the lower and upper confidence bounds of the forecast, respectively.

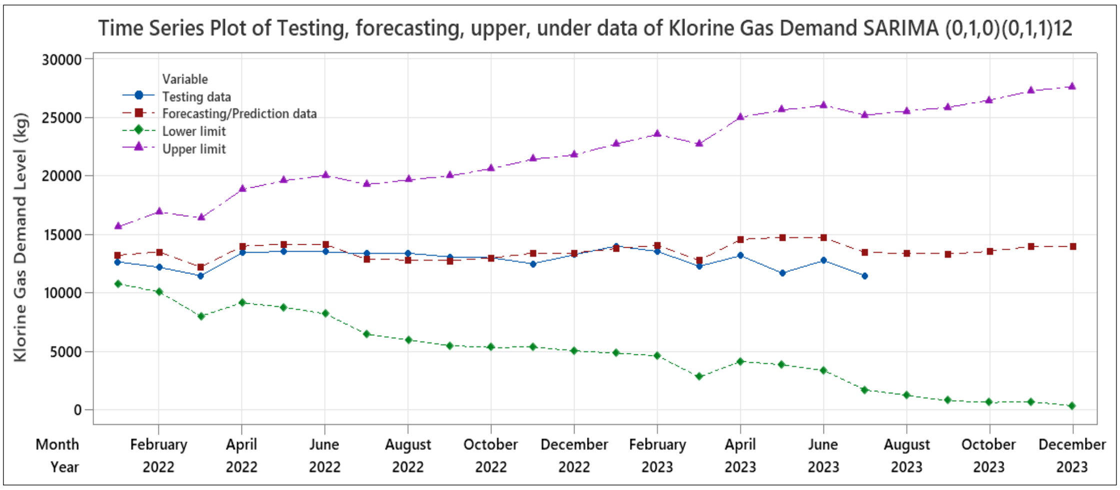

Figure 3. Graph of testing data and forecast data from August 2023 to December 2023

3.2. Chlorine gas demand distribution testing

The probability distribution of annual chlorine gas demand data from 2016 to 2023 was evaluated using the kolmogorov–smirnov and cramér–von mises tests, as defined in (8) and (10). These tests were used to verify that the data distribution aligns with the model assumptions employed in the analysis. The results of these tests provide a strong basis for determining the most representative distribution type for the chlorine gas demand data. A summary of the test results is presented systematically in Table 2 to facilitate interpretation and further discussion.

Table 2. Goodness-of-fit of probability distributions to chlorine gas demand data

No	Types of probability distributions	*KSp_ value*	*KS_ value*	*CvMp_ value*	*CvM_ value*	AIC	BIC	Values of distribution parameters	Mean $\mu = E(x)$	Decision
1	Normal	0.711	0.229	0.729	0.077	200.03	200.19	*Shape* = 0 *Scale* (σ) = 50,656.55 *Loc* (μ) = 124,626.93	124,626.93	H_0 Accepted
2	Gamma	0.739	0.224	0.669	0.087	202.67	202.92	*Shape* (λ) = 2.35 *Scale* (θ) = 52,987.19 *Loc* = 0	124,626.93	H_0 Accepted
3	Exponential	0.077	0.427	0.077	0.309	206.41	206.57	*Scale* (β) = 114,790.18 *Loc* (γ) = 9,836.75	124,626.93	H_0 Accepted
4	Erlang	0.234	0.345	0.084	0.371	207.07	207.31	*Shape* (k) = 2.35 *Scale* (δ) = 52,987.19 *Loc* = 0	124,626.93	H_0 Accepted

Table 2 presents the probability distributions, along with their shape, scale, and location parameters, that fit the chlorine gas demand data. Based on the kolmogorov–smirnov test for the probability distributions of the received demand data, the gamma distribution showed the highest *p*-value (0.739) and the smallest KS statistic (0.234), indicating the best fit among the other distributions. Meanwhile, the exponential distribution has the weakest p-value (0.077), approaching rejection. Based on the cramér–von mises test, which is used to measure the fit between the empirical distribution function and the theoretical distribution by considering the overall deviation, the Normal probability distribution yields the best results compared to the other distributions, with the highest *p*-value (0.729) and the smallest KS statistic (0.077). Compared to the kolmogorov–smirnov test, the cramér–von mises test exhibits more consistent sensitivity across the entire distribution [29].

Furthermore, according to [30], [31], the normal and gamma distributions yield relatively lower AIC and BIC values than other distributions. The difference in AIC and BIC values between the normal and gamma distributions is relatively small (ΔAIC = 2.64) and (ΔBIC = 2.73), with differences less than 3 (ΔAIC ≤ 3) and (ΔBIC ≤ 3); therefore, the models are considered equivalent. Compared to the exponential and erlang distributions, the differences in AIC and BIC values exceed 3, providing strong evidence that the normal and gamma distributions are better models. Therefore, it can be concluded that the normal and gamma distributions are the most appropriate probability distributions to represent the demand data distribution.

3.3. Inventory model of exponential EOQ, gamma, and erlang

Once the demand probability distributions were obtained, mathematical models were developed for the gamma, exponential, and erlang distributions that fit the chlorine gas demand data. The formulation of the reorder point (*r*) and the number of inventory shortages (*N*) for chlorine gas under an exponential probability distribution is developed through in (5) and (6), as follows:

$$\int_r^\infty \frac{1}{\beta} e^{-\frac{(x-\gamma)}{\beta}} dx = \frac{Qh}{C_u D} = \alpha$$
$$\Leftrightarrow r = [-\ln(\alpha)\beta] + \gamma \tag{11}$$

and

$$N = \int_r^\infty (x-r) f(x) dx = \int_r^\infty (x-r) \frac{1}{\beta} e^{-\frac{(x-\gamma)}{\beta}} dx \tag{12}$$

In a similar manner, the formulations for *r* and *N* in the model with gamma-distributed chlorine gas demand are derived as (13) and (14):

$$r = \frac{sc.gammaincinv[\lambda,(1-\alpha)]}{1/\theta} = \theta(sc.gammaincinv[\lambda,(1-\alpha)]) \tag{13}$$

and the N formulation is:

$$N = \int_r^\infty (x-r) f(x) dx = \int_r^\infty (x-r) \frac{\left(\frac{1}{\theta}\right)^\lambda x^{\lambda-1} exp\left(-\left(\frac{1}{\theta}\right)x\right)}{\Gamma(\lambda)} dx \tag{14}$$

Similarly, the development of the model with erlang-distributed demand includes formulations in which r and N are defined as (15) and (16):

$$r = \left[Sc.Gammaincinv\left(k, \frac{(1-\alpha)}{\Gamma(k)}\right)\right] / (1/\delta) \tag{15}$$

$$N = \int_r^\infty (x-r) \frac{(1/\delta)^k x^{k-1} e^{-(1/\delta)x}}{(k-1)!} dx \tag{16}$$

with k: shape parameters and δ: scale parameters.

Furthermore, the formulation for determining the optimal order quantity (Q) for chlorine gas is as (17):

$$Q = \sqrt{\frac{2D(A + C_u N)}{h}} \tag{17}$$

In this context, D represents the average annual demand, A denotes the ordering cost, C_u is the shortage cost, and N is adjusted based on the respective probability distribution formulation. The optimal solution is determined using the Hadley-Within algorithm [28], [32]. The parameter values of the probability distributions presented in Table 1 are employed to compute the model's optimal solution.

3.4. Model comparison

The parameters used in the model to obtain the optimal solution include an ordering cost (A_2) of IDR 5,000, an annual contract cost (A_1) of IDR 51,000,000, a unit purchase cost (p) of IDR 21,534.59 per kilogram, a holding cost (h) of IDR 2,153.46, and a shortage cost (C_u) of IDR 22,611.32. The optimal solution results for each probabilistic inventory model assuming demand follows normal, exponential, gamma, and erlang distributions are summarized in Table 3.

Table 3. Probabilistic optimum solution model (Q, r)

Variables	Chlorine gas demand probability distribution			
	Normal	Exponential	Gamma	Erlang
$Tc(Q,r)$ (IDR)	2,738,398,167.74	2,745,485,750.30	2,703,564,125.32	2,771,498,783.03
Iteration	3	9	7	5
r (kg)	1,486.69	4,052.53	2,777.85	77.07
Q (kg)	834.01	1,559.65	1,129.34	17,046.50
N (kg)	0.05	0.71	0.27	110.80
α (%)	0.00067	0.00119	0.00086	0.01303
ss (kg)	841.37	3,407.20	2,132.52	0
η (%)	99.99	99.89	99.96	83.77
O_s (IDR)	2,709,861.99	9,016,591.19	5,808,288.07	18,354,471.26
O_b (IDR)	2,683,789,840.51	2,683,789,840.51	2,683,789,840.51	2,683,789,840.51
O_p (IDR)	51,747,150.16	51,399,535.54	13,301,767.48	51,036,554.99
O_k (IDR)	2,709,861.99	9,016,591.19	664,229.26	18,317,916.27
AIC	200.03	206.41	202.67	207.07
BIC	200.19	206.57	202.92	207.31

Table 3 presents the optimal solutions of the probabilistic inventory model based on normal, exponential, gamma, and erlang probability distributions. The results indicate that the gamma distribution-based model for chlorine gas demand yields the optimal inventory policy compared to models using other probability distributions, as it has the minimum total cost and relatively small AIC and BIC values. The gamma distribution is most appropriate for chlorine gas demand data because it can describe right-skewed distributions and captures more complex variability, which the normal, erlang, and exponential distributions cannot. The minimum total cost obtained from the (Q, r) inventory model with the gamma distribution is $Tc(Q,r)_{gamma}$ = IDR 2,703,564,125.32, which is lower than the total costs associated with the normal, exponential, and erlang models. The reorder point for chlorine gas, denoted by r, is r_{gamma} = 2,777.85 kg, with an optimal order quantity of Q_{gamma} = 1,129.34 kg. These computations were performed using Python programming language with the assistance of the SymPy, SciPy, and math libraries

The comparative results of other inventory-related variables are presented graphically in Figure 4. Figure 4(a) illustrates that the erlang model yields the lowest reorder point value (r), while the exponential model yields the highest. As shown in Figure 4(b), the erlang model produces the largest order quantity (Q), whereas the normal model results in the smallest. Figure 4(c) indicates that the highest number of shortages occurs under the erlang model. Figure 4(d) presents a comparison of storage costs, ordering costs, and shortage costs across all models. Among the four models, the erlang model consistently incurs the highest values for these three cost components, while the gamma model generally exhibits the lowest. Specifically, the gamma model offers the lowest ordering cost, whereas the erlang model has the highest storage and shortage costs.

The gamma, normal, and exponential inventory models yield relatively similar inventory policies, characterized by relatively high reorder points (r) and moderate order quantities (Q). In contrast, the erlang model demonstrates a distinct inventory policy, notably lacking a safety stock (ss) (i.e., the safety stock value is zero), while exhibiting a low reorder point and a significantly large order quantity. The reorder point in the erlang inventory model is very small, even though the order quantity is very large; inventory shortages can also be significant, resulting in increased shortage costs. The gamma, exponential, and normal models tend to yield better inventory policies than the erlang model, thereby reducing the likelihood of chlorine gas shortages. Among the four models, the gamma model produces the most optimal inventory policy in terms of minimizing total cost.

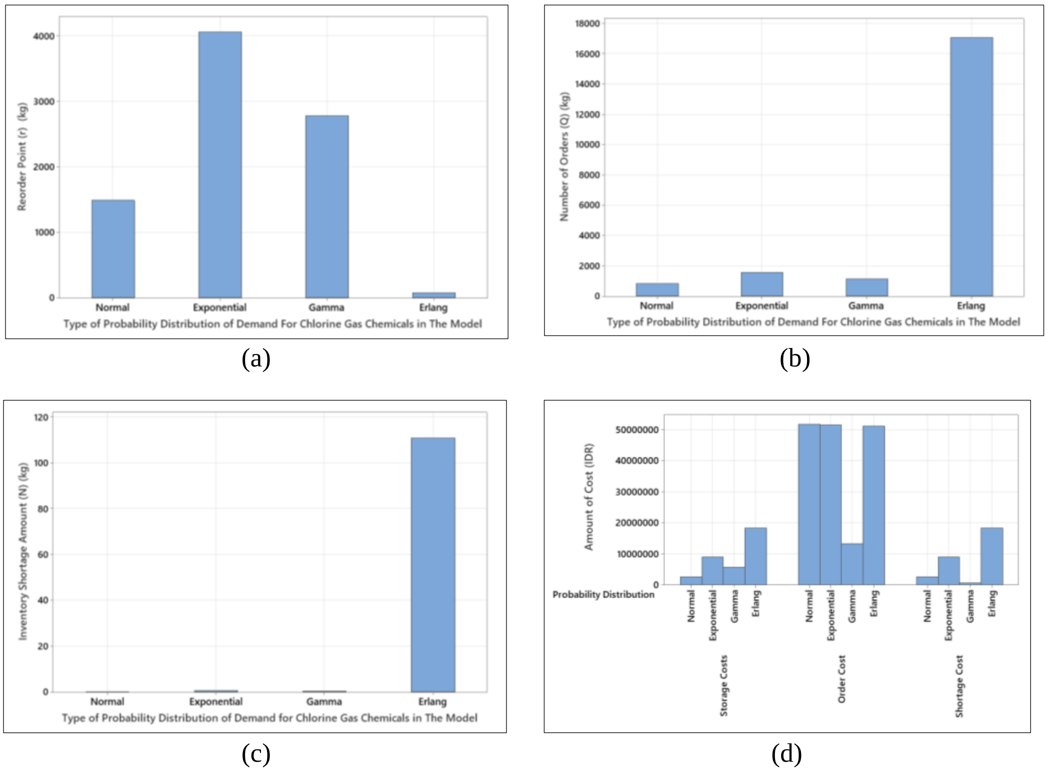

Figure 4. Comparison chart of variables based on the probability distribution of demand: (a) reorder point (r), (b) number of orders (Q), (c) inventory shortage amount (N), and (d) storage, order, shortage cost

4. CONCLUSION

A comparative analysis of inventory models based on probability distributions that fit the demand data pattern for chlorine gas namely, the normal, exponential, gamma, and erlang distributions shows that the gamma-based model yields a more optimal solution than the others. Low values of the AIC and BIC further reinforce the superior performance of the gamma model. Based on the results of this study, PDAM is advised to use the gamma probabilistic inventory model (Q,r), with an ordering policy when inventory reaches 2,777.85 kg and a purchase quantity of 1,129.34 kg. It is hoped that the PDAM company will conduct annual demand forecasting and evaluate the probability distribution of chlorine gas demand to optimize chlorine gas procurement and minimize costs. Future research could expand the scope of this study by incorporating additional factors that influence inventory models, such as variability in lead times, delivery delays, chemical expiration dates, and chemical depreciation, as well as by developing combined probabilistic inventory models for various types of chemicals. Additionally, other optimization methods, such as modern data-driven approaches or intelligent approaches (e.g., machine learning-based inventory policies), could also be applied. This could also be followed by the development of a highly useful inventory information system.

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AUTHOR CONTRIBUTIONS STATEMENT

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Evi Yuliza			✓	✓	✓	✓		✓		✓		✓	✓	✓

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

The authors state no conflict of interest.

INFORMED CONSENT

We have obtained informed consent from all individuals and companies involved in the study.

ETHICAL APPROVAL

The research related to human use has been complied with all the relevant national regulations and institutional policies in accordance with the tenets of the Helsinki Declaration and has been approved by the authors' institutional review board or equivalent committee; or: The research related to animal use has been complied with all the relevant national regulations and institutional policies for the care and use of animals.

DATA AVAILABILITY

The data that support the findings of this study are available from PDAM. Restrictions apply to the availability of these data, which were used under license for this study. Data are available from the authors with the permission of PDAM.

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