Quasi linear and stone geary utility functions based-internet service financing scheme with marginal costs and monitoring costs

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ABSTRACT

The use of computer network technology is currently increasing, especially on the internet network. To connect to the actual internet, it is a task for internet service provider (ISP). Providing advantages to ISPs, it requires a financing scheme. This study’s goal is to present a modified model for internet service financing schemes, within the customer choices and consumer satisfaction levels to maintain the schemes. To achieve the best outcomes, this updated model is built through marginal costs and cost monitoring while taking into account service quality based on stone-geary utility functions and quasi-linear utility functions. This research provides a solution regarding the differences in increasing consumer interest with payment options on model modification that will be provided. Traffic Digilib in a local server in Palembang. According to this study, a usage-based financing strategy and a two-part pricing of IDR 2727.8 per kbps will yield the highest revenues.

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1. INTRODUCTION

The use of computer network technology is currently increasing, especially in internet networks, including for internet of things (IoT) [1], [2]. To connect to our internet network we must subscribe to an internet service provider (ISP) [3]. ISP is the company that operates in the internet services sector [2]. In providing benefits, for ISPs, need a financing scheme where the scheme provided can guarantee the satisfaction of service providers, and service users [4], [5]. There are three information service financing schemes, namely flat fee, usage-based, and two-part tariff [6], [7].

In the issue of information service financing schemes, the utility function is one of the functions that can be applied to the problem of financing schemes [8], [9]. The quasi-linear utility function and the Stone-Geary utility function were employed in this study to gauge the degree of consumer satisfaction [5], [10]–[12]. In increasing the level of consumer satisfaction with the use of information services, apart from the utility function, it is necessary to increase marginal costs [13]–[17] and monitoring costs [18]–[21]. A marketing strategy is needed that will make the price cheaper than the total price of the packets so that it attracts consumer interest and is provided with payment options [22].

Based on this background, it is important to discuss the design of a customer preference-based internet service financing scheme [23]–[26]. The customer preference-based internet service financing scheme model is completed differentially [27]. Much research on pricing the information service, only stresses out optimization model and ignores the analytical approach [28]. So, in this discussion, the models...
are proved by the series of lemmas. By utilizing the functions to measure satisfaction, and make use of marginal costs, and monitoring costs, there are different schemes, and the models are observed. By customer preference, the modification model is applied to high-end and low-end as well as to heterogeneous consumers of high demand and low demand [29].

2. RESEARCH METHOD

The measures taken to complete this study are as follows:
- Describe digilib traffic data on the local server. Data was obtained from secondary data starting from January 1, 2022 to January 31, 2022 which is grouped into peak hours from 07.00 AM to 5.00 PM Indonesian time and non-peak hours from 5.01 PM to 06.59 AM.
- Define the parameters such as utility function in busy and non-busy hours, peak-time prices provided by ISPs, fees are required when following the services provided, the greatest degree of consumer in utilizing the service at peak and off-peak times, marginal and monitoring costs, consumer interest and payment options.
- Define the variables such as, service consumption levels during peak and off-peak hours, willingness to subscribe of the customer, consumer’s peak and off peak hour service consumption rates, and consumer’s decision-making factor regarding participation.
- For differential solution, determine the internet service financing scheme model based on the quasi-linear utility function and the Stone-Geary utility function with the addition of marginal costs and monitoring costs as well as consumer interest, payment options, optimization consumer problems and supplier’s optimization problems in flat fee, usage-based and two-part tariff financing schemes for heterogeneous consumers.
- Apply the modified model for the internet service financing scheme obtained from steps 5 by applying it to local server data already processed in step 1 and step 2.
- Complete the model in step 5 until the optimal solution is obtained. The optimization solution was assisted with LINGO 13.0 software.
- Compare the results from step 6 to obtain the optimal financing scheme for each type of consumer.

3. RESULTS AND DISCUSSION

The quasi-linear utility function and the stone-geary utility function, along with additional expenses for monitoring, consumer interest, and payment alternatives, are used in this chapter’s discussion of the modification model.

3.1. Formulation of parameters dan variables

The parameters and design variables are presented in Tables 1 and 2, as follows. Tables 3 and 4 basically describe the parameters set up for the model. In Table 3 and 4, $g_1/\gamma_1$ and $g_2/\gamma_2$ are service constants during peak hours whereas $h_1/\beta_1$ and $h_2/\beta_2$ are service constants during non-peak hours where the value of $g$ and $h$ are determined on condition of $g_1$, $g_2$, $h_1$, and $h_2$ positive integers, and $g_1 > h_1, g_2 > h_2, g_1 > g_2$ and $h_1 > h_2$ for a diverse group of high-end and low-end customers. The values of $g$ is for heterogeneous consumers with high-demand and low-demand $h$ is determined under the condition $g_1, g_2, h_1$ and $h_2$ positive integers and $g > h$, $g_1 = g_2 = g$ and $h_1 = h_2 = h$.

<table>
<thead>
<tr>
<th>Table 1. Parameters of each financing model</th>
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<tbody>
<tr>
<td>Parameters of each modification model</td>
</tr>
<tr>
<td>$U(E_d, W_n)$</td>
</tr>
<tr>
<td>$D_e$</td>
</tr>
<tr>
<td>$D_w$</td>
</tr>
<tr>
<td>$a$</td>
</tr>
<tr>
<td>$b_a$</td>
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<tr>
<td>$W'_a$</td>
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<td>$c$</td>
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<td>$x$</td>
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<tr>
<td>$y$</td>
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<tr>
<td>$z$</td>
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</tbody>
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\[
\begin{align*}
E_a & \quad \text{Service consumption levels during peak hours.} \\
W_a & \quad \text{Service consumption levels during off-peak hours.} \\
I_a, I'_a & \quad \text{If a customer decides not to subscribe, a variable with a value of 1 indicates that they do not want to do so and consumer a’s decision-making factor regarding participation, respectively.} \\
E'_a & \quad \text{Consumer a’s peak hour service consumption rate.} \\
W'_a & \quad \text{Consumer a’s use of services at off-peak hours.}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Table 2. Variables of each financing model</th>
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<tbody>
<tr>
<td>Variables of each modification model</td>
</tr>
<tr>
<td>(E_a)</td>
</tr>
<tr>
<td>(W_a)</td>
</tr>
<tr>
<td>(I_a, I'_a)</td>
</tr>
<tr>
<td>(E'_a)</td>
</tr>
<tr>
<td>(W'_a)</td>
</tr>
</tbody>
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\[
\begin{align*}
\text{Table 3. Parameter values of redesigned models} \\
\text{Parameters} & \quad \text{Flat fee} & \quad \text{Usage-based} & \quad \text{Two-part tariff} \\
E_1 & 25.94742 & 25.94742 & 25.94742 \\
E_2 & 11.22755 & 11.22755 & 11.22755 \\
W_1 & 10.95491 & 10.95491 & 10.95491 \\
W_2 & 9.95225 & 9.95225 & 9.95225 \\
c & 0 < c < 10 & 0 < c < 10 & 0 < c < 10 \\
t & 0 < t < 10 & 0 < t < 10 & 0 < t < 10 \\
p = \text{q} & 1 & 1 & 1 \\
x = y & 0.01 & 0.01 & 0.01
\end{align*}
\]

<table>
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<tr>
<th>Table 4. Parameter values for heterogeneous consumers</th>
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<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>(g_{1}/y_1) &amp; 4 &amp; 3</td>
</tr>
<tr>
<td>(g_{1}/y_2) &amp; 3 &amp; 3</td>
</tr>
<tr>
<td>(h_{1}/\beta_1) &amp; 2 &amp; 2</td>
</tr>
<tr>
<td>(h_{2}/\beta_2) &amp; 2 &amp; 2</td>
</tr>
</tbody>
</table>

3.2. Heterogeneous consumers modification model for quasi-linear utility function

For the financing plan with flat fees: Optimization of the customer’s issue will be as follows:

\[
\begin{align*}
\text{Max} & \quad g_a E_a + f(W_a) - D_E E_a - D_W W_a - D I_a - (E_a + W_a) c - (E_a + W_a) x - (E_a + W_a) y \\
\text{subject to} & \quad E_a \leq \bar{E} I_a, W_a \leq \bar{W} I_a
\end{align*}
\]

Optimization of the supplier problem will be as follows:

\[
\begin{align*}
\text{Max} & \quad p(D_E E'_1 + D_W W'_1 + D I'_1) + q(D_E D'_2 + D_W W'_2 + D I'_2) \\
\text{subject to} & \quad E_a \leq \bar{E} I_a, W_a \leq \bar{W} I_a \\
& \quad g_a E_a + f(W_a) - D_E E_a - D_W W_a - D I_a \geq 0, I_i = 0 \text{ or } 1
\end{align*}
\]

for financing plans based on consumption and two-part tariffs. Then, the optimization of the customer’s issue is as follows:

\[
\begin{align*}
\text{Max} & \quad g_a E_a + f(W_a) - D_E E_a - D_W W_a - D I_a - (c + t) E_a - (c + t) W_a - (x + y) E_a - (x + y) W_a \\
\text{subject to} & \quad E_a \leq \bar{E} I_a, W_a \leq \bar{W} I_a \\
& \quad g_a E_a + f(W_a) - D_E E_a - D_W W_a - D I_a \geq 0, I_i = 0 \text{ or } 1
\end{align*}
\]

Optimization of supplier problem is as follows:

\[
\begin{align*}
\text{Quasi linear and stone geary utility functions based-internet service} \quad \ldots \quad \text{(Indrawati)}
\end{align*}
\]
\[
\begin{aligned}
\text{Max} \quad & p(D_E E_1^a + D_W W_1^a + DI_1^a) + q(D_E E_2^a + D_W W_2^a + DI_2^a) \\
\text{Subject to} \quad & E_a \leq \bar{E}a, W_a \leq \bar{W}a \\
& g_a E_a + f(W_a) - D_E E_a - D_W W_a - DI_a \geq 0, I_i = 0 \text{ or } 1
\end{aligned}
\]

With \((E_a^*, W_1^*, I_1^*) = \arg\max g_a E_a + f(W_a) - D_E E_a - D_W W_a - DI_a\)

### 3.2.1. High-end and low-end heterogeneous consumers' modified models

Case 1a: If the ISP employs a financing method with a flat cost, it \(D_E = 0, D_W = 0, \) and \(D > 0\).

When the ISP’s price has no bearing on whether or not a customer uses a service during peak or off-peak hours, the user selects the maximum consumption level of \(E_1 = \bar{E}, W_1 = \bar{W}, E_2 = \bar{E}, \) and \(W_2 = \bar{W}\).

Maximize the function on consumer problem optimization so that it is obtained \(D = g_a \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y\). Thus, each high-end heterogeneous consumer is charged no more than \(g_1 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y, \) and low-end heterogeneous consumers no more than \(g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y\).

Case 1a uses a flat-cost financing plan to balance \(D\) for both categories of consumers. If stipulated \(g > g_2\), then the price for the costs of low-end heterogeneous consumers follows, resulting in the provision of high-end heterogeneous consumer costs \(g_1(p) < g_2(p + q) \iff g_1 < \frac{g_2(p + q)}{p}\) meaning if the consumer is charged \(g_1 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y\) then the cost for serving low-end heterogeneous consumers is followed by the price for serving high-end heterogeneous consumers so that.

The price for the costs of low-end heterogeneous consumers is then followed by the price for the provision of high-end heterogeneous consumers costs so that \(g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y, \) subsequently, both categories of customers can use the service. In this case for the optimization of the supplier’s problems namely.

\[
\begin{aligned}
\text{Max} \quad & (GH_1^a) + y(GH_2^a) = p(g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y) + q(g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y) \\
& (p + q)(g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y)
\end{aligned}
\]

Lemma 1a: If the ISP employs a flat fee financing method, the cost is:

\[
D = g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y
\]

with the maximum profit obtained is:

\[
(p + q)(g_2 \bar{E} + f(\bar{W}) - (\bar{E} + \bar{W})c - (\bar{E} + \bar{W})x - (\bar{E} + \bar{W})y)
\]

Case 2a: If the ISP uses a usage-based financing scheme, it is set \(D_E > 0, D_W > 0, \) and \(D = 0\) then, the problem of optimization for high-end heterogeneous consumers. Functions on the optimization of consumer problems into \(\text{Max} g_1 E_1 + f(W_1) - D_E E_1 - D_W W_1 - (c + t)E_1 - (c + t)W_1 - (x + y)E_1 - (x + y)W_1. \) To maximize the function of optimizing heterogeneous consumer problems, the high-end is carried out by the differentiation of \(E_1\) and \(W_1\), provided that \(\frac{\partial F}{\partial E_1} = 0\) and \(\frac{\partial F}{\partial W_1} = 0\),

\[
\frac{\partial F}{\partial E_1} = 0 \iff g_1 - (c + t) - (x + y) = D_E \iff E_1^* = \bar{E}
\]

and

\[
\frac{\partial F}{\partial W_1} = 0 \iff f'(W_1) - (c + t) - (x + y) = D_W \iff W_1^* = \bar{W}
\]

Problem of optimization for low-end heterogeneous consumers. The objective function on the optimization of consumer problem is then:

\[
\text{Max} g_2 E_2 + f(W_2) - D_E E_2 - D_W W_2 - (c + t)E_2 - (c + t)W_2 - (x + y)E_2 - (x + y)W_2
\]
To maximize the function of optimizing heterogeneous consumer problem, the low-end is carried out by the differentiation of $E_2$ and $W_2$, provided that $\frac{\partial F}{\partial E_2} = 0$ and $\frac{\partial F}{\partial W_2} = 0$.

$$\frac{\partial (g_2 E_2 + f(W_2) - D_2 E_2 - D_2 W_2 - \langle c+t \rangle E_2 - \langle c+t \rangle W_2)}{\partial E_2} = 0 \iff g_2 - \langle c+t \rangle - \langle x+y \rangle = D_2 \iff E_2^* = \bar{E}$$

and

$$\frac{\partial (g_2 E_2 + f(W_2) - D_2 E_2 - D_2 W_2 - \langle c+t \rangle E_2 - \langle c+t \rangle W_2)}{\partial W_2} = 0 \iff f'(W_2) - \langle c+t \rangle - \langle x+y \rangle = D_W \iff W_2^* = \bar{W}$$

Optimization of supplier problems will be:

$$\max \left( D_2 E_2^* + D_W W_2^* \right) + q(D_2 E_2^* + D_W W_2^*) = p(D_2 \bar{E} + D_W \bar{W}) + q(D_2 \bar{E} + D_W \bar{W}) = p(g_1 \bar{E} + \bar{W} f'(\bar{W}) - (c+t)\bar{E} - (c+t)\bar{W} - (x+y)\bar{E} - (x+y)\bar{W})$$

The ISP must maximize the objective function when applied to issues during peak hours $D_2$ and hence the best price $D_2$ cannot be greater than $g_1 - (c+t) - (x+y)$. If the ISP sets the price below $g_2 - (c+t) - (x+y)$, the profit is not at its best. Applied to problems at off-peak hours, the most affordable pricing $D_W \leq f'(W_2) - (c+t) - (x+y)$. If the ISP sets the price below $f'(W_2) - (c+t) - (x+y)$, then the profit is not optimal when $W_2 \leq \bar{W}$ and $E_2 \leq \bar{E}$. So, the best $D_W$ price is $f'(W_2) - (c+t) - (x+y)$. Thus, the optimal price given for peak hours is $D_W = a_2 - (c+t) - (x+y)$ and the optimal price given for non-peak hours is $D_W = f'(\bar{W}) - (c+t) - (x+y)$, the maximum profit is $(p+q) \left( g_2 \bar{E} + \bar{W} f'(\bar{W}) - (c+t)\bar{E} - (c+t)\bar{W} - (x+y)\bar{E} - (x+y)\bar{W} \right)$.

### Lemma 5a:

The ideal price charged during peak hours if the ISP implements a usage-based financing system is $D_W = a_2 - (c+t) - (x+y)$ and in non-peak hours is $D_W = f'(\bar{W}) - (c+t) - (x+y)$ with the maximum profit obtained being $(p+q) \left( g_2 \bar{E} + \bar{W} f'(\bar{W}) - (c+t)\bar{E} - (c+t)\bar{W} - (x+y)\bar{E} - (x+y)\bar{W} \right)$.

### Lemma 2a:

If the ISP uses a two-part tariff financing scheme, it $D_2 > 0$, $D_W > 0$, and $D > 0$. It is set by $g_1 > g_2$ then it can be assumed that $g_1(p) < g_2(p+q)$ $\iff$ $g_1 < \frac{g_2(p+q)}{p}$ means that if the consumer is charged $D_2 = a_1 - (c+t) - (x+y)$, $D_W = f'(W_2) - (c+t) - (x+y)$, and $D = f(\bar{W}) - \bar{W} f'(\bar{W})$ then only affluent, diverse consumers can use this service. If a customer gets billed by $D_2 = g_2 - (c+t) - (x+y)$, $D_W = f'(W_2) - (c+t) - (x+y)$, and $D = f(\bar{W}) - \bar{W} f'(\bar{W})$ then heterogeneous high-end and low-end consumers can follow the service. Optimization of supplier problems into $\max \left( D_2 E_2^* + D_W W_2^* + D_1^2 \right) + q(D_2 E_2^* + D_W W_2^* + D_1^2) = (p+q) \left( g_2 \bar{E} + \bar{W} f'(\bar{W}) - (c+t)\bar{E} - (c+t)\bar{W} - (x+y)\bar{E} - (x+y)\bar{W} \right)$.

### Lemma 3a:

If the ISP employs a two-part tariff financing plan, the ideal $D_2$ and $D_W$ will be $D_2 = g_2 - (c+t) - (x+y)$, $D_W = f'(W_2) - (c+t) - (x+y)$ and $D = f(\bar{W}) - \bar{W} f'(\bar{W})$ with the maximum profit obtained is $(p+q) \left( g_2 \bar{E} + \bar{W} f'(\bar{W}) - (c+t)\bar{E} - (c+t)\bar{W} - (x+y)\bar{E} - (x+y)\bar{W} \right)$.

### 3.3.2. Modification models for high-demand and low-demand heterogeneous consumers

The next lemma was discovered using similar evidence for the following three lemmas.

### Lemma 4a:

If the ISP employs a flat rate financing method, the cost $g_2 \bar{E} + f(\bar{W}_2) - (\bar{E}_2 + \bar{W}_2) c - (\bar{E}_2 + \bar{W}_2) x - (\bar{E}_2 + \bar{W}_2) y$ with the maximum profit obtained is $(p+q) \left( g_2 \bar{E} + f(\bar{W}_2) - (\bar{E}_2 + \bar{W}_2) c - (\bar{E}_2 + \bar{W}_2) x - (\bar{E}_2 + \bar{W}_2) y \right)$.

### Lemma 5a:

If the ideal price during peak hours if the ISP implements a usage-based financing system is $D_2 = g - (c+t) - (x+y)$ and the optimal price in non-peak hours is $D_W = f'(W_2) - (c+t) - (x+y)$ with the maximum profit obtained $\left( p+q \left( g_2 \bar{E} + \bar{W}_2 f'(\bar{W}_2) - (c+t)\bar{E}_2 - (c+t)\bar{W}_2 - (x+y)\bar{E}_2 - (x+y)\bar{W}_2 \right) \right)$.

### Lemma 6a:

If the ISP uses a two-part tariff financing arrangement $D_2 = g - (c+t) - (x+y)$, $D_W = f'(W_2) - (c+t) - (x+y)$ and $D = f(\bar{W}_2) - \bar{W}_2 f'(\bar{W}_2)$ with the maximum profit obtained is $p \left( g_2 \bar{E}_2 + \bar{W}_2 f'(\bar{W}_2) - (c+t)\bar{E}_2 - (c+t)\bar{W}_2 - (x+y)\bar{E}_2 - (x+y)\bar{W}_2 \right) + q \left( g_2 \bar{E}_2 + f(\bar{W}_2) - (c+t)\bar{E}_2 - (c+t)\bar{W}_2 - (x+y)\bar{E}_2 - (x+y)\bar{W}_2 \right)$.

### 3.4. Modified heterogeneous consumers model used for stone-gauery utility function

For the flat fee financing scheme, the optimization of consumer problems will be:

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Quasi linear and stone geary utility functions based-internet service ... (Indrawati)
Max $E_a, W_a, I_a (E_a - y_2)\beta_a + (W_a - y_2)\beta_a - D_E E_a - D_W W_a - D I_a - (E_a + W_a)c - (E_a + W_a)x - (E_a + W_a)y$

subject to $E_a \leq \bar{E} I_a, W_a \leq \bar{W} I_a$.

$(E_a - y_2)\beta_a + (W_a - y_2)\beta_a - D_E E_a - D_W W_a - D I_a - (E_a + W_a)c - (E_a + W_a)x - (E_a + W_a)y \geq 0$

$I_a = 0 \text{ or } 1$

Optimization of supplier problems is as follows:

Max $\sum_{D_D E_D W} p(D_E E_1^* + D_W W_1^* + D I_1^*) + q(D_E E_2^* + D_W W_2^* + D I_2^*)$

with $(E_a^*, W_a^*, I_a^*) = \arg \max (E_a - y_2)\beta_a + (W_a - y_2)\beta_a - D_E E_a - D_W W_a - D I_a$.

subject to $E_a \leq \bar{E} I_a, W_a \leq \bar{W} I_a, (E_a - y_2)\beta_a + (W_a - y_2)\beta_a - D_E E_a - D_W W_a - D I_a \geq 0, I_a = 0 \text{ or } 1$.

To finance usage-based and two-part tariff schemes then optimization of customer issues is as follows:

Max $\sum_{D_D E_D W} (E_a - y_2)\beta_a + (W_a - y_2)\beta_a - D_E E_a - D_W W_a - D I_a - (c + t)E_a - (c + t)W_a - (x + y)E_a - (x + y)W_a$

subject to $E_a \leq \bar{E} I_a, W_a \leq \bar{W} I_a, (E_a - y_2)\beta_a + (W_a - y_2)\beta_a - D_E E_a - D_W W_a - D I_a \geq 0, I_a = 0 \text{ or } 1$.

Optimization of supplier problems will be:

Max $\sum_{D_D E_D W} (E_a - y_2)\beta_a + (W_a - y_2)\beta_a - D_E E_a - D_W W_a - D I_a - (c + t)E_a - (c + t)W_a - (x + y)E_a - (x + y)W_a$

subject to $E_a \leq \bar{E} I_a, W_a \leq \bar{W} I_a, (E_a - y_2)\beta_a + (W_a - y_2)\beta_a - D_E E_a - D_W W_a - D I_a \geq 0, I_a = 0 \text{ or } 1$.

3.4.1 High-end and low-end heterogeneous consumers’ modified models

Using related proofs for the subsequent three lemmas.

Lemma 1b: if an internet service provider (ISP) adopts a flat rate financing method, the cost to users becomes $G = (E - y_2)\beta_2 + (W - y_2)\beta_2 - (E + W)c - (E + W)x - (E + W)y$ and the maximum profit earned is $(p + q)(\beta_2 (E - y_2)\beta_2 - (E + W)c - (E + W)x - (E + W)y) + \beta_2 (E - y_2)\beta_2 - (E + W)c - (E + W)x - (E + W)y$.

Lemma 2b: the ideal price is if the ISP employs a usage-based financing system $D_E = \beta_2 (E - y_2)\beta_2 - (E + W)c - (E + W)x - (E + W)y$ and $D_W = \beta_2 (W - y_2)\beta_2 - (E + W)c - (E + W)x - (E + W)y$.

Lemma 3b: the maximum profit obtained is $(p + q)(\beta_2 (E - y_2)\beta_2 - (E + W)c - (E + W)x - (E + W)y) + \beta_2 (W - y_2)\beta_2 - (E + W)c - (E + W)x - (E + W)y$.

3.4.2 Modification models in heterogeneous consumers of high demand and low demand

Using the same supporting evidence for the subsequent three lemmas then we have as follows.

Lemma 4b: the fee paid becomes a flat fee if the ISP adopts this type of financing $D = (\bar{E}_2 - y)\beta_2 + (\bar{W}_2 - y)\beta_2 - (\bar{E}_2 + \bar{W}_2)c - (\bar{E}_2 + \bar{W}_2)x - (\bar{E}_2 + \bar{W}_2)y$ and the maximum profit obtained is $(p + q)(\beta_2 (E - y)\beta_2 + (W - y)\beta_2 - (E + W)c - (E + W)x - (E + W)y)$.

Lemma 5b: the ideal price is if the ISP employs a usage-based financing scheme $D_E = \beta_2 (E - y)\beta_2 - (E + W)c - (E + W)x - (E + W)y$, and $D_W = \beta_2 (W - y)\beta_2 - (E + W)c - (E + W)x - (E + W)y$.

Lemma 6b: the ideal price is if the ISP employs a two-part tariff financing arrangement $D_E = \beta_2 (E - y)\beta_2 - (E + W)c - (E + W)x - (E + W)y$. The maximum profit is $\beta_2 (E - y)\beta_2 - (E + W)c - (E + W)x - (E + W)y$. The maximum profit is $\beta_2 (W - y)\beta_2 - (E + W)c - (E + W)x - (E + W)y$. The maximum profit is $\beta_2 (E - y)\beta_2 - (E + W)c - (E + W)x - (E + W)y$.
obtained is \[ p((E_2 - \gamma)^{\beta} + (W_2 - \gamma)^{\beta} + \beta(E_2 - \gamma)^{\beta-1}E_1 + \beta(W_2 - \gamma)^{\beta-1}W_1 - \beta(E_2 - \gamma)^{\beta-1}E_2 - \\
\beta(W_2 - \gamma)^{\beta-1}W_2 - (c + t)E_2 - (c + t)W_2 - (x + y)E_2 - (x + y)W_2 + q((E_2 - \gamma)^{\beta} + (W_2 - \gamma)^{\beta} - \\
(c + t)E_2 - (c + t)W_2 - (x + y)E_2 - (x + y)W_2). \]

3.5. Optimal financing scheme for heterogeneous consumers

The following are the results obtained based on calculations performed for a diverse group of consumers. Based on Table 5, the maximum profit of the two utility functions is in the stone-geary utility function with a profit of IDR 2727.77269 per kbps and profit of IDR 1685.95079 per kbps for high-demand and low-demand customers.

Table 5. Maximum advantages for high-end and low-end and high-demand and low-demand heterogeneous consumers

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Quasi-linear High-end and low-end</th>
<th>Stone-geary</th>
<th>Quasi-linear High-demand low-demand</th>
<th>Stone-geary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat fee</td>
<td>393.490</td>
<td>1177.515</td>
<td>264.18807</td>
<td>115.39062</td>
</tr>
<tr>
<td>Usage-based</td>
<td>632.772</td>
<td>2727.772</td>
<td>461.85804</td>
<td>1685.95079</td>
</tr>
<tr>
<td>Two-part tariff</td>
<td>392.752</td>
<td>327.25309</td>
<td>485.88746</td>
<td></td>
</tr>
</tbody>
</table>

4. CONCLUSION

Based on the findings and subsequent discussion, it can be concluded that a usage-based financing structure combined with a two-part tariff of IDR 2727.77269 per kbps yields the highest profit from the customer preference-based internet service financing scheme model for heterogeneous consumers (high end and low end as well as high and low demand). Basically, flat fee and two-part tariff schemes yield slightly different objective function values for all schemes. For further research, it is suggested to focus on the improvement of models (lemma by also taking care of consumers’ ability) to automatically transform into another scheme as they prefer to.

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REFERENCES


Quasi linear and stone geary utility functions based-internet service ... (Indrawati)


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